

Regression orthogonal experimental design and application in engineering technology

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Abstract. Regression orthogonal experimental design is the direct coupling of orthogonal experiment and regression analysis, and is a method in which the selection of the optimal scheme in the orthogonal experiment will be extended by the use of regression model. This paper introduces the research and development of orthogonal experimental design, and then gives the basic principle of regression orthogonal experimental design, the coding and tabulation, and scheme selection. At last, this paper describes the application in practical engineering design.

Introduction

Experiments in natural sciences and engineering technology are planned practices. Only scientific experimental design can use less number of experiments to achieve the desired goal of the experiment in a short time; otherwise, a lot of experimental data will be waste. Only a reasonable analysis and processing of experimental data can obtain the variation of studies, in order to achieve the purposes of production and scientific research.

In the 1940s, Japanese statisticians Dr. Genichi Taguchi used orthogonal table to arrange experiment. This method is simple, since then orthogonal design has been generally promoted the worldwide use. However, the optimal scheme obtained from orthogonal experimental design is limited to only a certain level, but not the optimal solution within a certain experimental range. Thus, the regression orthogonal experimental design which is direct coupling of orthogonal experimental design and regression analysis can use reasonable experimental design, fewer numbers of experiments to establish effective mathematical models.

Orthogonal experimental design

Orthogonal experimental design is the method by use of orthogonal arrangement, and orthogonal table is basis. Orthogonal table is design form of rules, with the expression of $L_n(a^p)$. Where, L is the code for the Orthogonal table, n is the number of experiments, a is the number of levels, p is the number of columns, which is the largest factor in the number of possible arrangements. Orthogonal table has the following two characteristics: neat comparability and uniformly dispersion. For example by orthogonal table $L_9(3^4)$, 9 experimental points distributed in three-dimensional space is shown in Fig. 1.

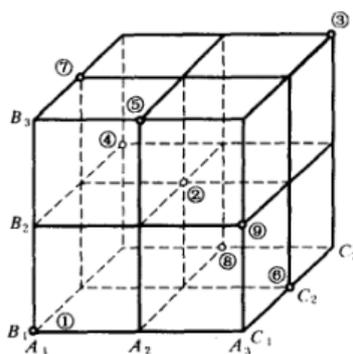


Fig. 1 Distribution of 9 experimental points in Orthogonal Table

Basic principle of regression orthogonal experimental design

However, the optimal scheme obtained from orthogonal experimental design above is limited to only a certain level, but not the optimal solution within a certain experimental range. Regression analysis is an effective method of data processing. By regression equations determined, the experimental result can be predicted and optimized. But regression analysis is often only used to handle and analyze experimental data passively, and does not involve the experimental design. If the advantages of both can be united, there are not only reasonable experimental design and fewer number of experiments, but also effective mathematical model established as expected. Regression orthogonal experimental design is such experimental design methods, which can select appropriate experimental points within the experimental range of factors, establish a high precision, good statistical property of the regression equation with fewer experiments, and solve the optimization problem of experiments.

The unitary regression orthogonal design is based on the principle of regression orthogonal design to establish the unitary a regression equation between experimental indicator y and experimental factors x_1, x_2, \dots, x_m :

$$\hat{y} = a + b_1x_1 + b_2x_2 + \dots + b_mx_m$$

Or

$$\hat{y} = a + \sum_{j=1}^m b_jx_j + \sum b_{kj}x_{kj}, \quad k = 1, 2, \dots, m-1 (j \neq k)$$

Determine the range of factors

According to the experimental indicator y , m factors x_1, x_2, \dots, x_m are selected to investigate, and the range of each factor is determined. The range of factor x_j is $[x_{j1}, x_{j2}]$, x_{j1} is the upper level of factor x_j , and x_{j2} is the lower level. Their arithmetic average is

$$x_{j0} = (x_{j1} + x_{j2})/2$$

Where x_{j0} is the zero level of factor x_j .

The difference between the upper level and the zero level is the spacing change of factor x_j , denoted by Δ_j , namely:

$$\Delta_j = x_{j2} - x_{j0} \text{ or } \Delta_j = (x_{j2} - x_{j1})/2$$

Code the level of factors

Encoding is carried linear transformation for each level of x_j , namely:

$$z_j = (x_j - x_{j0})/\Delta_j$$

Where, z_j is the encoding to x_j , they are one to one.

Obviously, x_{j1}, x_{j2}, x_{j3} are encoding respectively to -1,0,1, as $z_{j1} = -1, z_{j2} = 0, z_{j3} = 1$. Usually x_j is called natural variable, and z_j is specification variable. The coding result can be expressed as

Table 1:

Table 1 Factors coding table

z_j	x_j			
	x_1	x_2	...	x_m
lower level(-1)	x_{11}	x_{21}	...	x_{m1}
upper level(1)	x_{12}	x_{22}	...	x_{m2}
zero level(0)	x_{10}	x_{20}	...	x_{m0}
spacing change Δ_j	Δ_1	Δ_2	...	Δ_m

The purpose of encoding is to make "equal" to each level in the coding space is, that the range of z_j is [-1, 1], and will not be affected by the natural unit and the value of the variable. So encoding can make the regression problems with y and x_j converted into regression results of y , thus the regression calculation is simplified greatly.

The unitary regression orthogonal design table

For unitary regression orthogonal experimental design, it is important to select a suitable orthogonal table, which is generally determined by the specific number of factors and interactions. For example, for two factor levels, level of 1 and 2 are converted to the corresponding coding -1 and 1. For the three level of factors, it is need to make level of 1, 2, 3 in the table converted into -1, 0, 1 respectively. As example $L_9(3^4)$, the conversion result is as follows:

Table 2 the factors encoding conversion table of $L_9(3^4)$

Experiment Number	z_1	z_2	z_3	z_4
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1	-1	-1	-1	-1
2	-1	0	0	0
3	-1	1	1	1
4	0	-1	0	1
5	0	0	1	-1
6	0	1	-1	0
7	1	-1	1	0
8	1	0	-1	1
9	1	1	0	-1

Establish unitary regression equation

The key of regression equation is to determine the regression coefficients. Let the total number of experiments is n , including m_c times m levels of experimentation and m_0 times and zero level experiment, namely: $n = m_c + m_0$. Since this experiment does not involve zero level experiment, then $n = m_c$.

If the results are $y_i (i = 1, 2, \dots, n)$, according to two features of the principle of least squares regression and orthogonal table, the coefficients of a linear regression equation can be calculated as follows:

$$a = \frac{1}{n} \sum_{i=1}^n y_i = \bar{y}$$

$$b_j = \frac{1}{m_c} \sum_{i=1}^n z_{ji} y_i, j = 1, 2, \dots, m$$

$$b_{kj} = \frac{1}{m_c} \sum_{i=1}^n (z_k z_j)_i y_i, j > k, k = 1, 2, \dots, m - 1$$

In the above formula, z_{ji} represents the encoding of levels of the z_j series, $(z_k z_j)_i$ represents the encoding of levels of the $z_k z_j$.

Application of regression orthogonal experimental design in engineering design

In order to determine the more significant factors on the air supply airflow and their degree of importance, this paper proposes the design and calculation method of the air supply airflow, the applies orthogonal experiment for experimental arrangement, and while uses the regression orthogonal for conclusion.

Orthogonal experimental design for air flow organization

In the study, due to many factors and the larger changes in the value of factors, if a comprehensive experiment was chosen, there will be large amount of experiments and results, so orthogonal experiment design is used. If the 1 experiments are arranged by orthogonal experiments

design, the amount of experiments results will reduce and the analysis process will be simplified, and while the accuracy and reliability of the experimental results can not be affected.

In the methods of evaluation on objectively reflect the body of the thermal environment, the average temperature of the human skin (t_{sk}) is a very effective and objective measure of human thermal sensation, which will change with environmental conditions change. In this paper, t_{sk} is chosen as a temperature environment.

During the experiment, in order to highlight the influence of various factors to air flow organization and avoid interference with other external conditions, indoor temperature, humidity and other conditions are fixed, and the changing of four factors associated with the air is focused on to study the role and impact of the airflow. Looking at the data, the following four factors are identified: the way of delivery outlet, the location of delivery outlet, the velocity of supply air and the temperature of supply air. These four factors are important factors affecting indoor air-conditioned environment in air distribution. In order to determine the impact of human comfort properties by the environment on their common role, human physiology experiments are carried out to determine the results of specific impact. The table of specific level of factors is below:

Table 3 the table of specific level of factors

Level	Factor			
	the way of delivery outlet	the location of delivery outlet(m)	the velocity of supply air(m/s)	the temperature of supply air($^{\circ}C$)
1	Round straightening vane	0.4	0.25	18
2	Swirl	0.7	0.5	20
3	DY swirl inducible	1.0	1.0	22

Experimental program by $L_9(3^4)$ is below:

Table 4 the experimental program by $L_9(3^4)$

Level	Way(A)	Location(B)	Velocity(C)	Temperature(D)
1	1(Round straightening vane)	1(0.4)	1(0.25)	1(20)
2	1(Round straightening vane)	2(0.7)	2(0.5)	2(22)
3	1(Round straightening vane)	3(1.0)	3(1.0)	3(24)
4	2(Swirl)	1(0.4)	2(0.5)	3(24)
5	2(Swirl)	2(0.7)	3(1.0)	1(20)
6	2(Swirl)	3(1.0)	1(0.25)	2(22)
7	3(DY swirl inducible)	1(0.4)	3(1.0)	2(22)
8	3(DY swirl inducible)	2(0.7)	1(0.25)	3(24)
9	3(DY swirl inducible)	3(1.0)	2(0.5)	1(20)

The intuitive analysis and analysis of variance of orthogonal experimental results

In order to reduce errors and make results more reasonable, experiment was repeated taken, and the two experiments with the same level are performed. After the actual measurement, the data obtained as shown in Table 5.

Table 5 the table of experimental test results

Level	Way (A)	Location (B)	Velocity (C)	Temperature (D)	Skin temperature difference 1	Skin temperature difference 2
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1	1	1	1	1	3.89	3.8
2	1	2	2	2	2.92	2.95
3	1	3	3	3	2.51	2.56
4	2	1	2	3	2.8	2.78
5	2	2	3	1	3.6	3.6
6	2	3	1	2	2	2.03
7	3	1	3	2	4.38	4.29
8	3	2	1	3	2.09	2.1
9	3	3	2	1	2.83	2.8
\bar{K}_1	6.21	7.31	5.30	6.84		
\bar{K}_2	5.60	5.75	5.69	6.19		
\bar{K}_3	6.16	4.91	6.98	4.95		
R	0.61	2.40	1.68	1.89		

The intuitive analysis of orthogonal experimental results

From table 4, the skin temperature difference in the 6th experiment $A_2B_3C_1D_2$ is the smallest, but the 6th experiment is not necessarily the optimal experiment in comprehensive experiments.

$\bar{K}_1, \bar{K}_2, \bar{K}_3$ are the average of the corresponding experimental indicators of levels for each column. Since \bar{K}_2 is the smallest value to column A, so 2 is the optimal level for factor A. Similarly, the level for other three factors can be drawn, and the final calculated combination is $A_2B_3C_1D_1$.

In accordance with the principle the bigger the R , the greater the impact, the importance of factors influence sorted in descending is B, C, D, A .

By data in Table 4, the horizontal trend graph is draw to observe trend of indicators.

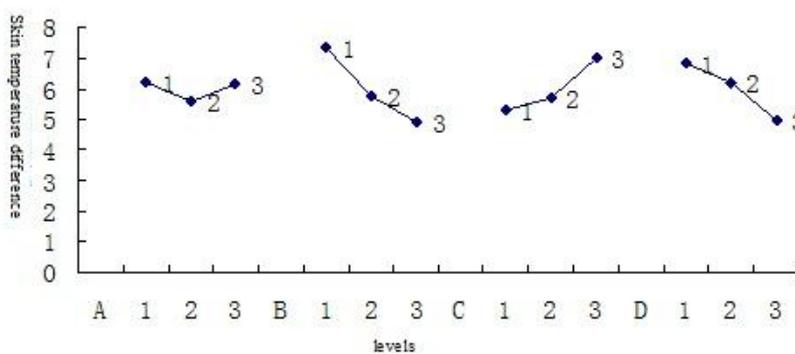


Fig. 2 the figure of intuitive analysis

From Fig. 2, the temperature difference changes with each factor changing trends. (1) Skin temperature difference first decreases and then increases with the change of the way of delivery outlet. (2) Skin temperature difference decreases by the location of delivery outlet, and the magnitude of decreases is smaller and smaller. (3) Skin temperature difference increases by the velocity of supply air, and the magnitude of increases is bigger and bigger. (4) Skin temperature difference decreases by the temperature of supply air, and the magnitude of decreases is bigger and bigger.

The analysis of variance of orthogonal experimental results

Table 6 ANOVA of experimental test results

Source	SS	df	MS	F	Sig.
B	4.46	2	2.23	12.64	Significant
C	2.31	2	1.15	6.54	Influential
D	2.78	2	1.39	7.87	Influential
Error (A)	0.35	2	0.04		

From Table 6, for B, the value of F is 12.64, and the impact on the temperature difference is the most significant. For D, the value of F is 7.87, and the impact on the temperature difference is influential, and then the same to C. The Primary and secondary relationship of four factors is *B, C, D, A*, which is consistent with the previous analysis.

Regression analysis of the experimental results

Since the way of delivery outlet is not an easy quantifiable indicators, so this indicator can be analyzed separately, as a categorical variable. The remaining three factors change with the level, and the experimental index varies regularly, because these three factors equally spaced levels, so that they can establish multiple linear regression equation, then the function shows the relationship between factors and experimental index.

According to the formula in front, three regression equations can be obtained as follows:

For round straightening vane:

$$\hat{y}_1 = 3.105 - 0.437x_2 - 0.437x_3 - 0.437x_4 \quad (1)$$

For Eq. 1, while $x_2 = 0.4$, $x_3 = 0.25$, $x_4 = 20$, the absolute value of temperature difference obtains the minimum $5.92^{\circ}C$, which can not make the temperature difference within an appropriate range. So the design of round straightening vane for air-conditioning is not reasonable.

For swirl:

$$\hat{y}_2 = 20802 - 0.258x_2 + 0.528x_3 - 0.37x_4 \quad (2)$$

For Eq. 2, while $x_2 = 0.4$, $x_3 = 1.0$, $x_4 = 20$, the absolute value of temperature difference obtains the minimum $2.17^{\circ}C$, which can make the temperature difference within an appropriate range. So the design of swirl for air-conditioning is better than round straightening vane.

For DY swirl inducible:

$$\hat{y}_3 = 3.081 - 0.207x_2 + 0.747x_3 - 0.24x_4 \quad (3)$$

For Eq. 3, while $x_2 = 0.4$, $x_3 = 1.0$, $x_4 = 20$, the absolute value of temperature difference obtains the minimum $1.17^{\circ}C$. So the design of DY swirl inducible for air-conditioning is the most suitable.

Conclusions

The application of orthogonal experiment not only reduces the amount of experiments, but also simplifies data analysis, does not affect the conclusions drawn, so orthogonal experiment can get good effect for saving manpower, material and time.

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