

A New Voltage Stability Control Model Considering Active Power Constraints on Weak Branches

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Abstract. A new voltage stability control model was presented to guarantee the static voltage stability of power system using the active power constraints of weak branches as the static voltage stability margin constraints. A local index for voltage stability was used to recognize the key contingencies, the weak branches and their maximum active power. Based on DC power flow equations, a static security analysis method was adopted to deduce the active power flow expressions on weak branches under every key contingency. According to the active power flow expressions and maximum active power on weak branches, the static voltage stability margin constraints were established. The proposed voltage stability control model was quadratic, and the predictor-corrector primal dual interior method was used to solve the proposed model. The simulations of the IEEE14-bus system and IEEE118-bus system proved that the proposed voltage stability control model was correct and effective.

Introduction

In recent years, voltage instability occurred in many power systems all over the world and resulted in power failure [1]. Thus, in order to relieve or at least minimize the system from the voltage instability problem, many electric utilities have made a great deal of effort in system studies related to voltage stability. Voltage stability mainly includes dynamic/transient voltage stability and static voltage stability. There are many research achievements on the assessment method for static voltage stability [1-4]. However, it is extremely important to use effective preventive control to improve the pre-contingency operating state of power system to guarantee the static voltage stability in various contingency conditions and stressed load condition.

The preventive control for the static voltage stability can mainly be formulated by the optimal power flow (OPF) models considering the static voltage stability margin constraint [5-8]. And there are two basic and complementary concepts for these models: linearization optimization models and nonlinear optimal models. References [5-7] proposed the linearization optimization models where the static voltage stability margin constraints were expressed by the linearization sensitivity of static voltage stability index with respect to control variables. Unfortunately, the power system is a nonlinear system and the nonlinear characteristic is predominant when it is unstable or close to collapse point. Therefore, a linearization model has limitations [9]. References [8] presented the nonlinear model in which the static voltage stability margin constraints were expressed by power flow equations with load parameter in normal operating condition, stressed load condition and multi-contingency conditions. This control model can reflect the nonlinear characteristic of power system. However, when a power system is very big or a large number of critical contingencies must

be considered, the number of static voltage stability margin constraints is extremely large and the preventive control model becomes very complicated. This results in difficulties in solving the model and even no feasible solution for the preventive control [10].

Actually, the static voltage instability generally originates from one or several weak branches whose active powers exceed their transfer capabilities. If the static voltage stability margin constraints can be expressed by the active power constraints of weak branches, the preventive control model will be greatly simplified. In order to achieve the goal, determining weak branches causing the static voltage stability problem is a crucial step. There are several localized line-based voltage stability indices which can identify weak branches [11,12]. Particularly, the voltage stability indices presented in References [12] can be used to estimate the maximum transfer capabilities of weak branches. If the active powers on weak branches exceed their maximum transfer capabilities, the static voltage stability problem occurs.

Based on the concept above, this paper presents a preventive control optimization model using the active power constraints of weak branches as static voltage stability margin constraints. A localized line-based voltage stability index is used to determine critical contingencies as well as corresponding weak branches and their transfer capabilities. A static security analysis based on DC power flow equations is used to obtain the quadratic expressions for active powers on weak branches in each critical contingency, which leads to a quadratic preventive control model. It has been proved that a quadratic optimization model is very efficient in computations when the predictor corrector primal dual interior point method (PCPDIPM) is used [13].

Formulation of the proposed preventive control model

The static voltage stability margin constraints for the proposed preventive control. Reference [12] proposed a static voltage stability index named the Extended Line Stability Index (ELSI). The ELSI of each line must be higher than 1.0 or equal to 1.0 to guarantee the static voltage stability of the system [12]. In practical operation, in order to avoid the system being operated near to the collapse point, a secure static voltage stability margin must be considered. This leads to the ELSI threshold. The threshold can be denoted by α and is little bit larger than 1.0.

When the ELSI is used to recognize the key contingencies and weak branches, the load flow in each contingency condition must be calculated. Then, the ELSI and actual active power on every line are calculated according to the load flow result. If ELSI of the line ij is lower than α under the contingency condition of line kl in outage, the contingency of line kl in outage is recognized as a key contingency, line ij is recognized as a weak branch whose maximum active power $P_{ij_weakmax}$ can be denoted by Equation (1).

$$P_{ij_weakmax} = ELSI_{ij_weak} \times P_{ij_weak} \quad (1)$$

Where, P_{ijmax} , P_{ij} , $ELSI_{ij}$ respectively represent the maximum active power, the actual active power, ELSI of line ij under the contingency condition of line kl in outage.

Using the static security analysis which is on the basis of DC power flow equations [14], the active power on weak branch ij , which is under the key contingency of line kl in outage, can be calculated by Equation (2).

$$P_{ij1}(e, f) = P_{ij0}(e, f) + \rho_{ij}^{kl} P_{kl0}(e, f) \frac{X_{kl}}{X_{ij}} \quad (2)$$

Where X_{ij} and X_{kl} respectively represent the reactance of line ij and line kl ; e , f respectively represent the real parts and imaginary parts of the bus voltage vectors in normal operating state; $P_{ij0}(e, f)$ and $P_{kl0}(e, f)$, which are the quadratic function of e and f under rectangular coordinate system, respectively represent the active power on line ij and line kl in normal operating state; $P_{ij1}(e, f)$ represents the active power of line ij in the key contingency on line kl in outage; ρ_{ij}^{kl} represents the transfer coefficient of line kl with respect to line ij . And in normal operating condition, the expression of $P_{ij1}(e, f)$ is same to $P_{ij0}(e, f)$ because the value of ρ_{ij}^{kl} is zero. In the key contingency condition, the computation of ρ_{ij}^{kl} can be referred to Reference [14].

According to Equation (1) and Equation (2), the active power constraints of weak branches in key contingencies can be established by Equation (3).

$$P_{ij0}(e, f) + \rho_{ij}^{kl} P_{kl0}(e, f) \frac{X_{kl}}{X_{ij}} \leq P_{ij_weak\ max} \quad (3)$$

Under any contingency condition, as long as the active power exceeds its maximum active power on one line, the static voltage instability will occur. Therefore, Equation (3) can be considered as the static voltage stability margin constraints of the proposed voltage stability control model because it denoted the active power constraints of weak branches.

Proposed quadratic optimization model for preventive control. After obtaining the static voltage stability constraint in critical contingencies, the proposed preventive control model can be established by Equations (4)-(16). In the proposed preventive control model, the objective is to minimize the load-shedding and the network active power loss. The unknown controllable variables to be optimized include the active power outputs P_G of generators, reactive power outputs Q_G of generators, reactive power injections Q_C of shunt capacitors, reactive power injections Q_R of shunt reactors, LTC (loading tap changers) turn ratios k and active load curtailments C . The unknown state variables to be optimized include the real parts e and imaginary parts f of bus voltages.

$$\min \quad \sum_{i \in N_B} P_{Gi} - \sum_{i \in N_B} (P_{Di} - C_i) + \sum_{i \in N_B} w_i C_i \quad (4)$$

$$s.t. \quad P_{Gi} - (P_{Di} - C_i) - \sum_{ij \in S_{Li}} P_{Lij}(e, f) - \sum_{ij \in S_{Ti}} P_{Tij}(e, f) = 0 \quad i = 1, \dots, N_B \quad (5)$$

$$Q_{Gi} + Q_{Ci} + Q_{Ri} - (Q_{Di} - C_i Q_{Di} / P_{Di}) - \sum_{ij \in S_{Li}} Q_{Lij}(e, f) - \sum_{ij \in S_{Ti}} Q_{Tij}(e, f) = 0 \quad i = 1, \dots, N_B \quad (6)$$

$$e_i f_m - e_m f_i = 0 \quad t = 1, \dots, N_T \quad (7)$$

$$e_i - k_t e_m = 0 \quad t = 1, \dots, N_T \quad (8)$$

$$V_{i\min}^2 \leq V_i^2 = e_i^2 + f_i^2 \leq V_{i\max}^2 \quad i = 1, \dots, N_B \quad (9)$$

$$k_{t\min} \leq k_t \leq k_{t\max} \quad t = 1, \dots, N_T \quad (10)$$

$$P_{Gi\min} \leq P_{Gi} \leq P_{Gi\max} \quad i = 1, \dots, N_G \quad (11)$$

$$Q_{Gi\min} \leq Q_{Gi} \leq Q_{Gi\max} \quad i = 1, \dots, N_G \quad (12)$$

$$Q_{Ci\min} \leq Q_{Ci} \leq Q_{Ci\max} \quad i = 1, \dots, N_C \quad (13)$$

$$Q_{Ri\min} \leq Q_{Ri} \leq Q_{Ri\max} \quad i = 1, \dots, N_R \quad (14)$$

$$0 \leq C_i \leq P_{Di} \quad i = 1, \dots, N_B \quad (15)$$

$$P_{ij0}(e, f) + \rho_{ij}^{kl} P_{kl0}(e, f) \frac{X_{kl}}{X_{ij}} \leq P_{ij_weak\ max} \quad ij \in S_{L_weak} \quad (16)$$

where N_B , N_G , N_C , N_R and N_T respectively represent the number of system buses, number of generator buses, number of shunt capacitor buses, number of shunt reactors buses and number of LTC branches; S_{Li} and S_{Ti} respectively represent the set of line branches and LTC branches connected to bus i ; S_{L_weak} represents the weak branches set. P_{Di} and Q_{Di} respectively represent the active and reactive power loads at bus i . C_i represents the active power load curtailment at bus i . The reactive power load curtailment at bus i is assumed to be proportional to C_i with a constant power factor, which is shown in Equation (6). w_i represents the weighting factor reflecting the importance of load at bus i ; the magnitudes of the weighting factors only need to be selected in a relative sense. (Note that every weighting factor is set to be 100 in the given examples in Section 4, which indicates equal importance for loads at each bus.)

Equations (5) and (6) respectively represent the equality constraints for the active and reactive power flows. In these equations, $P_{Lij}(e, f)$, $Q_{Lij}(e, f)$, $P_{Tij}(e, f)$ and $Q_{Tij}(e, f)$, which are quadratic functions of optimal variables e and f , respectively represent the active and reactive powers on line branch ij and LTC branch ij . And their expressions can be referenced to Reference [13]. Equations (7) and (8) respectively represent the voltage conversion relation of LTC branches, which are denoted in Reference [13].

Equations (9)-(15) respectively represent the constraints of the lower limits and upper limits for the voltage magnitudes at each bus, turn ratios of LTC, active power and reactive power outputs of generators, reactive power injections of shunt capacitor and shunt reactors, and active power load curtailments at each bus. Equation (16) is the static voltage stability margin constraints, which have been derived earlier in Equation (3).

Simulations

The basic data of the test systems. The correctness and effectiveness of the proposed preventive control is demonstrated using the simulations for the IEEE 14-bus system and IEEE 118-bus system. The following assumptions are made to ensure that the IEEE 14-bus system case and the IEEE 118-bus system case become possibly to loss static voltage stability in some contingency conditions.

- In the IEEE 14-bus system, the active load at bus 14 is increased to be 53.8MW, and the reactive load is increased with an assumption of a constant power factor.
- In the IEEE 118-bus system, the active loads at buses 43, 44 and 45 are respectively increased to be 6MW, 62MW, 140.45MW; and the reactive loads are increased with an assumption of a constant power factor.
- In the normal operating condition and any contingency condition, the threshold α of ELSI is set to be 1.1.

Results and analysis of simulation. Before the preventive control, the power flow is calculated and the static voltage stability is analyzed in normal operating state and each contingency state of the three test systems. The information of critical contingencies and the corresponding weak branches is shown in **Table 1**. The maximum transfer capacities of weak branches, which are shown in the fifth column, are determined according to Equation (1). The calculated results indicate the start points for the three test systems are in the insecure operation state since the active powers on weak branches in critical contingencies exceed their maximum transfer capabilities and the ELSI of

each weak branch is smaller than 1.1.

Table 1 Information of critical contingencies and weak branches before the proposed preventive control

Test system	Critical contingency /weak branch	Active power on weak branch (p.u.)	ELSI of weak branch	The maximum transfer capability of weak branch (p.u.)
IEEE14-bus system	branch1-2 in outage/branch 1-5	3.2754	1.0535	3.1369
	branch 9-14 in outage / branch 13-14	0.6460	1.0967	0.6441
IEEE118-bus system	branch 34-43 in outage / branch 43-44	0.3793	1.0607	0.3657
	branch 44-45 in outage / branch 43-44	0.6782	1.0499	0.6473
	branch 45-46 in outage / branch 45-49	2.0191	1.0552	1.9369

As mentioned in Introduction, in some nonlinear preventive control models, the power flow equality constraints with load parameter are respectively used as the static voltage stability margin constraints, which has limitation when the preventive control considers multi-contingency conditions. Here, the number of static voltage stability margin constraints in the proposed preventive control model is compared with the static voltage stability margin constraints mentioned above. The result is shown in **Table 2**. It can be seen that the number of constraints in the proposed model is far smaller than that of power flow equality constraints with load parameter, which can greatly reduce the size of the preventive control problem. This advantage will become more significant for a larger power system.

Table 2 Comparison for number of static voltage stability constraints in two preventive control models

Test system	The number of critical contingencies	The number of static voltage stability margin constraints	
		The active power constraints of weak branches	The number of power flow equality constraints with load parameter
IEEE14-bus system	2	2	56
IEEE118-bus system	3	3	708

After the first iteration of the proposed preventive control, the values of control variables are adjusted by the optimization model and the second contingency screening is performed. After the second contingency screening, the information of the critical contingencies and weak branches illustrated in **Table 1** is shown in **Table 3**. It can be seen from **Table 3** that the ELSI values of the weak branches become larger than 1.1 and the active powers on the weak branches become lower than their maximum transfer capabilities shown in **Table 1**. And after the second contingency screening, there is no critical contingency and weak branch for each of the three systems. This suggests that the system becomes secure from an insecure state through the proposed preventive control model since there is no violation from voltage stability.

For the three test systems, the entire preventive control process ends after the first iteration of the proposed preventive control. A few more iterations may be required for other systems. The other results of the entire preventive control for the three test systems are summarized in **Table 4**. The

iteration numbers in the PCPDIPM, active network power loss and load curtailment are also given in **Table 4**. It can be seen that the load curtailment is not required to ensure the static voltage stability margin in all contingency conditions for both the IEEE 14-bus and IEEE 118-bus system after the proposed preventive control. This demonstrates the effectiveness and correctness of the proposed preventive control model.

Table 3 Information of critical contingencies and weak branches illustrated in Tab.1 after the first iteration of the proposed preventive control

Test system	Critical contingency /weak branch	Active power on weak branch(p.u.)	ELSI of weak branch
IEEE14-bus system	branch 1-2 in outage / branch 1-5	3.1335	1.1091
	branch 9-14 in outage / branch 13-14	0.6409	1.1018
IEEE118-bus system	branch 34-43 in outage / branch 43-44	0.3624	1.1400
	branch 44-45 in outage / branch 43-44	0.6471	1.3585
	branch 45-46 in outage / branch 45-49	1.8999	1.1979

Table 4 Result of the entire preventive control process

Test system	Iteration number of PCPDIPM	Network active power loss (p.u.)	Load curtailment (p.u.)
IEEE14-bus system	15	0.5079	0.0000
IEEE118-bus system	12	1.3384	0.0000

Summary

Majority of preventive control models for static voltage stability that have been presented so far are based on the linearization assumption. A possible reason is the consideration in computing burdens. This paper proposed a new preventive control optimization model for static voltage stability with three features. Firstly, the proposed model can reflect the nonlinear characteristics of power system and overcome the limitations of linearization models. Secondly, the static voltage stability margin constraints are represented using the active power constraints only on weak branches which can be easily identified by a local voltage stability index. This greatly reduced the number of preventive control constraints since the number of weak branches causing voltage instability is always small in a real power system. Thirdly, the proposed model is expressed in a purely quadratic form which can be efficiently solved using the predictor-corrector primal dual interior method. The second and third features together can significantly reduce computing efforts.

The IEEE 14-bus system and IEEE 118-bus system are used as examples. The correctness and effectiveness of the proposed preventive control model are demonstrated by the simulation results of the test systems and verified by the results obtained from the continuation power flow method.

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