

## An Improved Neural Network Observer designed for nonlinear system

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**Abstract.** Considering the traditional linear observer works satisfactorily only in the neighborhood of the operating point and the conventional nonlinear observers depends on the precise model of the system, a non conventional Neural Network (NN) observer for nonlinear system is considered in this paper. The proposed neuro-observer is a three-layer feedforward neural network, which is trained extensively with the error backpropagation learning algorithm including a correction term to guarantee good tracking as well as bounded NN weights. The main problem of designing the neural network observer is that using artificial neural network to identify the nonlinear parts of the system, because the artificial neural network has good recognition abilities for nonlinear system. And then using a Luenberger observer to reconstruct the states of the system. Furthermore, the Lyapunov direct method is used in order to ensure the stability of the proposed non-conventional observer. The proposed observer is applied to 2 degrees of freedom horizontal manipulator to evaluate its performance. The simulation results show that the method is suitable for the low precision model of the nonlinear system and satisfy the requirement of the control.

### Introduction

The state observation problem is one of the essential issues in modern control theory since most control methodologies assume that the states are available for feedback. However in most cases, it is difficult to obtain the state variables for direct online measurements, in most realistic cases, only the outputs of the plant can be measured. Of course, using measuring tools to obtain more states is also a method. However, it is not economical to purchase many equipments and the extra measurement tools can reduce the precision of model and control qualities. Therefore, estimating the state variables by observers plays a crucial role in the control of processes to achieve better performances.

In order to realize the state feedback control system or other needs, D.G., R.W. and J.E. put forward the concept of state observer and state reconstruction method to solve the problem of the state unavailable<sup>[1]</sup>. Luenberger observer and Kalman filter are the most popular linear observers whose properties are well defined<sup>[2-4]</sup>. However, the locally linearized model has defectiveness and it works satisfactorily only in the neighborhood of the operating point. Therefore, several conventional nonlinear observers have been suggested during the past decades, such as high-gain observers<sup>[5]</sup>, sliding mode observers and others<sup>[6,7]</sup>. However these conventional methods are relatively complex and are applicable to systems with accurate model. Whereas the use of non conventional techniques as neural networks in many scientific disciplines mainly in identification and control of dynamical systems has been widespread<sup>[8,9]</sup>.

Neural network is a kind of soft computing methods<sup>[10,11]</sup>. Neural networks are flexible tools for time series processing and pattern recognition. Moreover, neural network techniques have showing a good promise as competitive methods for signal processing. So far, neural networks can be used to overcome these problems.

A non conventional Neural Network (NN) observer for nonlinear system is considered in this paper. The proposed neuro-observer is a three-layer feedforward neural network, which is trained extensively with the state estimation error backpropagation learning algorithm including a

correction term to guarantee good tracking as well as bounded NN weights. And a linear filter is added to the Sigmoid activation function to improve noise immunity. Furthermore, the Lyapunov direct method is used in order to ensuring the stability of the proposed non-conventional observer and of the NN weight errors. The proposed observer is applied to 2 degrees of freedom horizontal manipulator to evaluate its performance. The simulation results show the effectiveness of the proposed observer.

### The Proposed Neural Network Based Observer

The main problem of designing the neural network observer is that using artificial neural network to identify the nonlinear parts of the system, because artificial neural network has good recognition abilities for nonlinear system<sup>[12,13]</sup>. And then using a traditional state observer to reconstruct the state of the system. The goal of the design is that the neural network observer, which can estimate the states of the system accurately, and the error system is asymptotically stable.

Consider the general model of nonlinear system by the following equation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + g(x, u) \\ y &= Cx(t) \end{aligned} \quad (1)$$

Where,  $u \in \mathbb{R}^n$  and  $y \in \mathbb{R}^m$  are state variable, control input and output respectively.  $g(x, u)$  is unknown, nonlinear functions of state variables,  $x(t)$  is state vector and  $A$  is a Hurwitz matrix.

In order to facilitate the proof, following definitions are taken for the systems under consideration.

**Definition 1.**  $V$  is the weight between input node  $i$  and the hidden node  $j$ ,  $W$  is the weight between hidden node  $j$  and output node  $k$ , and  $\|W\| \leq W_M, \|V\| \leq V_M$ .

**Definition 2.**  $\sigma(Vz) = \frac{2}{1 + \exp[-(2\sum_j V_{ji}x_i)]} - 1$ ,  $\sigma(\cdot)$  is sigmoid function,  $\varepsilon(x)$  is The error of modeling and  $z = [x^T \ u^T]^T$ .

$g(x, u)$  is the nonlinear part of the system and it's a smooth function. Then there must exist weights and thresholds such that  $g(x, u)$  can be represented as :

$$g(x, u) = W\sigma(Vz) + \varepsilon(x) \quad (2)$$

Defining the estimation of  $g(x, u)$  by the linear in parameter neural network as:

$$\hat{g}(\hat{x}, u) = \hat{W}\sigma(\hat{V}\hat{z}) \quad (3)$$

with an observable  $(A, C)$  pair, the observer model can be selected as

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + L(y - C\hat{x}) + \hat{g}(\hat{x}, u) \\ \hat{y} &= C\hat{x} \end{aligned} \quad (4)$$

where,  $\hat{x}$  denotes the state observer.  $L$  is the observer gain.

Defining the state tracking error vector  $e$  as:

$$e = x - \hat{x}$$

than

$$\begin{aligned} \dot{e} &= \dot{x} - \dot{\hat{x}} = (A - LC)e + g(x, u) - \hat{g}(\hat{x}, u) \\ e_y &= Ce \end{aligned} \quad (5)$$

Taking Laplace transform for (5) :

$$se(s) = (A - LC)e(s) + L[g(x, u) - \hat{g}(\hat{x}, u)]$$

$$e_y(s) = C[sI - (A - LC)]^{-1} L[g(x, u) - \hat{g}(\hat{x}, u)]$$

$$e_y(s) = H(s)L[g(x, u) - \hat{g}(\hat{x}, u)] \quad , \quad (6)$$

where,  $H(s) = \frac{C}{[sI - (A - LC)]}$ .

By the formula (5), we have

$$\begin{aligned} \dot{e} &= \dot{x} - \dot{\hat{x}} = (A - LC)e + e_w \sigma(\hat{V}\hat{z}) + \Delta \\ e_y &= Ce \end{aligned} \quad (7)$$

Where,  $\Delta = W[\sigma(Vz) - \sigma(\hat{V}\hat{z})] + \varepsilon(x) \leq \bar{\Delta}$ ,  $e_w = W - \hat{W}$ .

By the formula (6) and (7)

$$e_y(s) = H(s)L(s)L^{-1}(s)[e_w \sigma(\hat{V}\hat{z}) + \Delta], \quad (9)$$

Where,  $H(s)L(s) = C_c(sI - A_c)^{-1}$ ,  $L(s)$  is the filter transfer function, and it has stable poles.

### Stability Analysis

**Theorem 1.** Consider the plant model (1) and the observer model (4). If the Neural network adaptive law is designed as:

$$\begin{aligned} \dot{\hat{W}} &= F_1 e (\sigma_1(\hat{V}\hat{z}))^T - \rho F_1 \|e\| \hat{W} \\ \dot{\hat{V}} &= (\hat{W} \sigma_2(\hat{V}\hat{z}))^T F_2 e \hat{z} - \rho F_2 \|e\| \hat{V} \end{aligned}$$

where  $\rho > 0$ ,  $F_1, F_2$  are Positive definite symmetric matrices,  $\sigma_1(\hat{V}\hat{z}) = L^{-1}(s) \cdot \sigma(\hat{V}\hat{z})$ ,  $\sigma_2(\hat{V}\hat{z}) = L^{-1}(s) \cdot \sigma(\hat{V}\hat{z})(1 - \sigma(\hat{V}\hat{z}))$ , than the error  $e$  is uniformly ultimately bounded, and the states of the system is bounded.

### Proof:

Consider the Lyapunov function candidate as

$$\mathcal{G} = \frac{1}{2} e^T P e + \frac{1}{2} \text{tr}(e_w^T F_1^{-1} e_w) + \frac{1}{2} \text{tr}(e_v^T F_1^{-1} e_v) \quad (10)$$

with  $P = P^T > 0$  satisfying  $G^T P + P G^T = -Q$ ,  $Q$  is a positive-definite matrix.

The time derivative of (10) is given by

$$\dot{\mathcal{G}} = \frac{1}{2} \dot{e}^T P e + \frac{1}{2} e^T P \dot{e} + \text{tr}(e_w^T F_1^{-1} \dot{e}_w) + \text{tr}(e_v^T F_1^{-1} \dot{e}_v) \quad (11)$$

$$\leq -\frac{1}{2}e^T Q e + e^T P [e_w \sigma_1(\hat{V}\hat{z}) + \Delta] + tr[\rho e_w^T \|e\| \hat{W} - e_w^T e (\sigma_1(\hat{V}\hat{z}))^T] + tr\{\rho e_v^T \|e\| \hat{V} - e_v^T F_2^{-1} [(\hat{W}\sigma_2(\hat{V}\hat{z}))^T F_2 e \hat{z}^T]\} \quad (12)$$

Based on the definition of the trace, we have

$$tr[e_w^T e (\sigma_1(\hat{V}\hat{z}))^T] = (\sigma_1(\hat{V}\hat{z}))^T e_w^T e \quad (13)$$

and 
$$tr[e_w^T (W - e_w)] \leq W_M \|e_w\| - \|e_w\|^2 \quad (14)$$

$$tr[e_v^T (V - e_v)] \leq V_M \|e_v\| - \|e_v\|^2 \quad (15)$$

Now by using (13), (14), (15) and (12), we can get

$$\begin{aligned} \dot{g} &\leq -\frac{1}{2}\lambda_{\min}(Q)\|e\|^2 + e^T P \Delta + e^T (P - I) e_w \sigma_1(\hat{V}\hat{z}) + \rho \|e\| (W_M \|e_w\| - \|e_w\|^2) \\ &\quad + \rho \|e\| (V_M \|e_v\| - \|e_v\|^2) - \text{sgn}(\hat{z})(\hat{W}\sigma_2(\hat{V}\hat{z}))^T F_2 e_v^T F_2^{-1} e \\ &\leq -\frac{1}{2}\lambda_{\min}(Q)\|e\|^2 + \|e^T P \Delta\| + \|e^T (P - I) e_w \sigma_1(\hat{V}\hat{z})\| + \rho \|e\| (W_M \|e_w\| - \|e_w\|^2) \\ &\quad + \rho \|e\| (V_M \|e_v\| - \|e_v\|^2) + \|e_v^T F_2^{-1} \sigma_m (W_m + \|e_w\|) F_2 e \hat{z}^T\| \end{aligned} \quad (16)$$

Extracting  $\|e\|$  from the formula (16), and Assuming  $\|\hat{z}^T\| \leq z_M$ , we have

$$\dot{g} \leq \|e\| \left\{ -\frac{1}{2}\lambda_{\min}(Q)\|e\| + \|P\|\bar{\Delta} + \|P\|\|e_w\|\sigma_m + \|e_w\|\sigma_m + \rho\left(\frac{W_M^2}{4} + \frac{V_M^2}{4}\right) + K_1\|e_v\|(W_M + \|e_w\|) \right\} \quad (17)$$

Where,  $K_1 = \|F_2^{-1}\| \|F_2\| \sigma_m \cdot z_M$ .

Defining,

$$\begin{aligned} \Omega &= \|(P - I) e_w \sigma_1(\hat{V}\hat{z})\| + \rho (W_M \|e_w\| - \|e_w\|^2) + \rho (V_M \|e_v\| - \|e_v\|^2) \\ &\quad + \|e_v^T F_2^{-1} \sigma_m (W_m + \|e_w\|) F_2 e \hat{z}^T\| \\ &\leq \|e_w\| [\|P - I\| \sigma_m + \rho W_M] - \rho \|e_w\|^2 - \rho \|e_v\|^2 + \|e_v\| (K_1 W_M + \rho V_M) \\ &\quad + K_1 \|e_v\| \|e_w\| \\ &= -\left(\rho + \frac{K_1}{2}\right) (\|e_w\| - \alpha)^2 - \left(\rho + \frac{K_1}{2}\right) (\|e_v\| - \beta)^2 + \left(\rho + \frac{K_1}{2}\right) (\alpha^2 + \beta^2) \end{aligned} \quad (18)$$

Where,

$$\alpha = \frac{\|P - I\| \sigma_m + \rho W_M}{2\rho + K_1}, \quad \beta = \frac{K_1 W_M + \rho V_M}{2\rho + K_1} \quad (19)$$

Hence, by using (18) the equality (17) becomes,

$$\begin{aligned} \dot{g} &\leq \|e\| \left\{ -\frac{1}{2}\lambda_{\min}(Q)\|e\| + \|P\|\bar{\Delta} - \left(\rho + \frac{K_1}{2}\right) (\|e_w\| - \alpha)^2 \right. \\ &\quad \left. - \left(\rho + \frac{K_1}{2}\right) (\|e_v\| - \beta)^2 + \left(\rho + \frac{K_1}{2}\right) (\alpha^2 + \beta^2) \right\} \end{aligned} \quad (20)$$

Notice that,

$$\begin{aligned}
 -(\rho + \frac{K_1}{2})(\|e_w\| - \alpha)^2 &\leq 0 \\
 -(\rho + \frac{K_1}{2})(\|e_v\| - \beta)^2 &\leq 0
 \end{aligned}$$

So the time derivative of  $\mathcal{G}$  is given by

$$\dot{\mathcal{G}} \leq \|e\| \left[ -\frac{1}{2} \lambda_{\min}(Q) \|e\| + \|P\| \bar{\Delta} + (\rho + \frac{K_1}{2})(\alpha^2 + \beta^2) \right] \quad (21)$$

To make  $\dot{\mathcal{G}} \leq 0$ , need to satisfy the following conditions

$$\|e\| \geq \frac{2[\|P\| \bar{\Delta} + (\rho + \frac{K_1}{2})(\alpha^2 + \beta^2)]}{\lambda_{\min}(Q)} \quad (22)$$

or

$$\begin{aligned}
 \|e_w\| &\geq \left\{ \frac{1}{2}(\alpha^2 + \beta^2) + \frac{\|P\| \bar{\Delta}}{2\rho + K_1} \right\}^{\frac{1}{2}} + \alpha \\
 \|e_v\| &\geq \left\{ \frac{1}{2}(\alpha^2 + \beta^2) + \frac{\|P\| \bar{\Delta}}{2\rho + K_1} \right\}^{\frac{1}{2}} + \beta
 \end{aligned} \quad (23)$$

So, we can see from the the formula (22) and (23) the error  $e$  is uniformly ultimately bounded, and the estimation error  $e_w$  and  $e_v$  of the neural network are uniformly ultimately bounded.

### Convergence Analysis of the System

The formula (7) can be rewritten as

$$\begin{aligned}
 \dot{e}_x &= A_c e_x + e_w \sigma(\hat{V}\hat{z}) + \Delta \\
 e_y &= C e_x
 \end{aligned} \quad (24)$$

where,  $A_c = A - LC$ . Defining  $e_u = e_w \sigma(\hat{V}\hat{z}) + \Delta$ , than

$$\begin{aligned}
 \dot{e}_x &= A_c e_x + e_u \\
 e_y &= C e_x
 \end{aligned} \quad (25)$$

Multipled by the  $e^{-A_c t}$  on both sides of (25), than

$$e^{-A_c t} (\dot{e}_x - A_c e_x) = \frac{d}{dt} (e^{-A_c t} \cdot e_x) \quad (26)$$

$$e^{-A_c t} (\dot{e}_x - A_c e_x) = \frac{d}{dt} (e^{-A_c t} \cdot e_x) = e^{-A_c t} \cdot e_u$$

We compute the integral for (26)

$$\int_0^t e^{-A_c \tau} (\dot{e}_x - A_c e_x) d\tau = \int_0^t \frac{d}{dt} (e^{-A_c \tau} \cdot e_x) d\tau = \int_0^t e^{-A_c \tau} \cdot e_u(\tau) d\tau \quad (27)$$

Thus,

$$e^{-A_c t} \cdot e_x(t) - e_x(0) = \int_0^t e^{-A_c \tau} \cdot e_u(\tau) d\tau \quad (28)$$

The solution of the nonhomogeneous equation is

$$e_x(t) = e^{A_c t} e_x(0) + e^{A_c t} \cdot \int_0^t e^{-A_c \tau} \cdot e_u(\tau) d\tau = e^{A_c t} e_x(0) + \int_0^t e^{A_c(t-\tau)} \cdot e_u(\tau) d\tau, \quad (29)$$

Where,

$$\|e_u\| = \|e_w \sigma(\hat{V}\hat{z}) + \Delta\| \leq \|e_w\| \sigma_M + \bar{\Delta} \quad (30)$$

Since the system error is bounded then the term  $e^{A_c t} e_x(0) + \int_0^t e^{A_c(t-\tau)} d\tau$  is bounded. According that the solution of the nonhomogeneous differential equation has the same form, we can write the state of the system as

$$x(t) = e^{A_c t} x(0) + \int_0^t e^{A_c(t-\tau)} B_u(\tau) d\tau, \quad (31)$$

Where,  $B_u(\tau) = W \sigma(Vz) + \varepsilon(x) \leq W_M \sigma_M + \varepsilon_M$ .

According to the analysis, the system status is bounded.

## Simulation

Using two joint manipulator system as the simulation object, the dynamic model are as follows ,

$$\begin{bmatrix} \dot{x}_{11} \\ \dot{x}_{12} \\ \dot{x}_{21} \\ \dot{x}_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{21} \\ x_{22} \end{bmatrix} + g(x_{11}, x_{12}, x_{21}, x_{22}) \quad (32)$$

Where,  $g(x_{11}, x_{12}, x_{21}, x_{22})$  is the nonlinear and uncertain function of the system, and  $x_{11} = q_1, x_{12} = \dot{q}_1, x_{21} = q_2, x_{22} = \dot{q}_2$  are the location and speed signals of joints 1 and 2.

Choose the observer parameters as

$$\rho = 0.001, L = \begin{bmatrix} 40 & 5 & 30 & 2 \\ 15 & 20 & 50 & 80 \end{bmatrix}, L(s) = \frac{1}{s+2.5}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

The observation results are shown in figure 2 and 3.

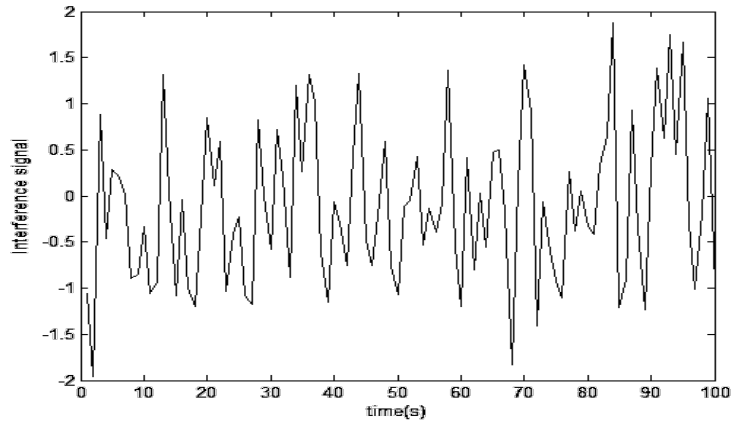


Fig.1 The random disturbance signals

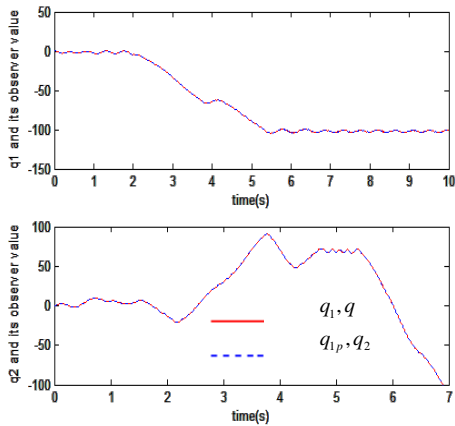


Fig. 2 Position signal  $q_1, q_2$  and the observed value  $q_{1p}, q_{2p}$

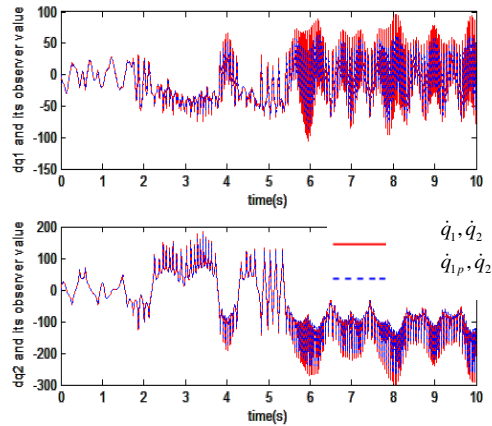


Fig. 3 Speed signal  $\dot{q}_1, \dot{q}_2$  and the observed value  $\dot{q}_{1p}, \dot{q}_{2p}$

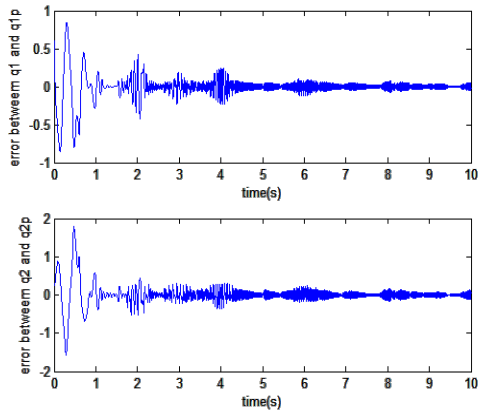


Fig. 3 The observation errors of  $q_1$  and  $q_2$

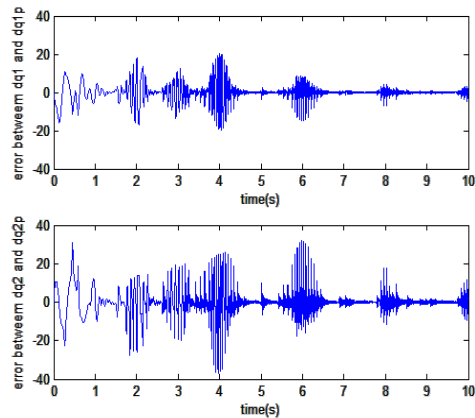


Fig. 4 The observation error of  $\dot{q}_1$  and  $\dot{q}_2$

## Conclusions

An improved Neural Network Observer for a general nonlinear system was considered in this paper. The structure of the neural network observer is simple, and do not need the observation object to have an accurate mathematical model. The Neural Network Observer is designed to solve the problem of the uncertainty system. It has been shown from the simulation results that the

proposed neural Network observer is efficient and permits the rapid reconstruction of the state variables of the two joint manipulator.

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