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Progress on the Behavior of Solutions to Three-dimensional Keller-Segel-Navier-Stokes System

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Abstract: The three-dimensional Keller-Segel-Navier-Stokes equation system, with its rich biological and physical backgrounds and practical significance, has attracted significant attention in the study of partial differential equations. A central focus of this research has been the investigation of the global existence and blow-up criteria of its solutions. While there have been numerous results on the global existence of solutions in two-dimensional spaces, numerous challenges remain in understanding the behavior of solutions in three dimensions. This article aims to analyze the progress made in studying the behavior of solutions to the three-dimensional Keller-Segel-Navier-Stokes equation system. Key areas of interest include the global existence of weak solutions, the long-time behavior of these weak solutions, and certain blow-up criteria for local strong solutions on bounded domains. Finally, we offer a prospective outlook on the research progress and future directions in understanding the behavior of solutions to the three-dimensional Keller-Segel-Navier-Stokes equation system, highlighting the remaining challenges and opportunities for further exploration

Keywords. Keller-Segel-Navier-Stokes system; weak solution; long-time behavior; local strong solutions; blow up criteria.

INTRODUCTION

With the development of modern science and technology, various issues related to partial differential equations have attracted widespread attention. Therefore, the application of partial differential equations to study various phenomena in physics, biology, chemistry, economics, and other fields has become an important research direction. We know that the study of many mathematical and physical problems can ultimately be attributed to the study of partial differential equations and systems of equations. Therefore, the related solution problems of partial differential equations and systems of equations are particularly important. The Keller-Segel system proposed by Keller and Segel in 1970 is an important mathematical model that describes biological phenomena, used to describe the movement state of biological populations, cell populations, or the evolution process of microorganisms. Biological mathematics has been continuously improved in a wide range of applications and now possesses a relatively complete theoretical basis, which has also promoted the development of mathematical theory. As a classic chemotaxis model, the Keller-Segel model has been widely studied by mathematicians and biologists. Chemotaxis refers to a biological characteristic in which cells and microorganisms have free movement from high-density to low-density directions under the effect of diffusion, and at the same time, they gather in high-density directions under the influence of chemical substances. Chemotaxis plays a crucial role in many complex biological processes such as pattern formation, bacterial aggregation, tumor invasion, angiogenesis, and has practical significance for its research. Named after Claude-Louis Navier and George Gabriel Stokes, the Navier-Stokes equation is a fundamental equation describing the motion of fluid media. By applying Newton's second law to fluid motion and assuming that the stress term of the fluid is the sum of dissipative terms (velocity gradients) and pressure, we can obtain the Navier-Stokes equation, which is a fundamental equation describing viscous fluids. Since the proposal of the Navier-Stokes equation nearly two hundred years ago, its importance has stemmed from its widespread applications in meteorology, ocean fluid dynamics, and air fluid dynamics around aircraft wings. On the other hand, the Navier-Stokes equation is also an important class of nonlinear partial differential equations, reflecting the mechanical laws of viscous fluid (also known as real fluid) motion. It has a wide range of applications in aviation dynamics, astrophysics, geomechanics, weather forecasting, oil and gas exploration, and information processing. As a fundamental equation in fluid mechanics, the existence and smoothness of solutions to the Navier-Stokes equation are one of the

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seven major mathematical problems of the millennium. The Keller-Segel-Navier-Stokes system of equations has a strong biological and physical background and is an extremely important set of partial differential equations in both application and theory. The study of its solution properties has always been an important topic in partial differential equations. Although the Keller-Segel-Navier-Stokes system of equations has been proposed for nearly sixty years, significant progress has only been made in the study of its solution properties in the past two decades. Currently, research results on the solution properties of the Keller-Segel-Navier-Stokes system of equations mainly focus on the two-dimensional case, while there are few known results for the three-dimensional case. The overall existence of weak solutions, the large-time behavior of weak solutions, and some blow-up criteria for local strong solutions of the three-dimensional Keller-Segel-Navier-Stokes system of equations on bounded domains are issues of great concern.

RELATED WORK

We consider the Keller-Segel-Navier-Stokes equations in R^N:

$$\begin{cases} \partial_t u - u\cdot\nabla u + \nabla\pi - \nabla u = n\nabla\phi, & (1) \\ div\, u = 0, & (2) \\ \partial_t n + u\cdot\nabla n - \nabla n + bn^2 - an = -\nabla\cdot(n\nabla p) - \nabla u\cdot(n\nabla q), & (3) \\ \partial_t p + u\cdot\nabla p - \nabla p = -np, & (4) \\ \partial_t q + u\cdot\nabla q - \nabla q + q = n \; in \; \mathbb{R}^N \times (0, \; \infty), & (5) \\ (u,n,p,q)(\cdot,0) = (u_0,n_0,p_0,q_0)(\cdot) \; in \; \mathbb{R}^N (N\geq 3). & (6) \end{cases}$$

Here, u represents the fluid velocity, π represents the pressure, n, p and q represent the concentrations of amoebae, oxygen, and chemical attractant, respectively. Potential q is a smooth function, a is a real constant, and b is a normal number. When u=0, the system of equations (3), (4), and (5) is the Keller-Segel system of equations [Keller and Segel, 1970; Keller and Segel, 1971; Keller and Segel, 1971]. The Keller-Segel model has been the focus of attention by many scientists and scholars as a classic chemotaxis model, and there have been many research results [Biler, 1999; Hillen and Painter, 2009; Wrzosek, 2006; Winkler, 2011]. When $\phi = 0$, the system of equations (1) and (2) is the famous Navier-Stokes system of equations. As the basic equation in fluid mechanics, the existence and smoothness of the Navier-Stokes equation solution are one of the seven major mathematical problems in the millennium. For nearly two centuries, the Navier-Stokes equations have been extensively studied by numerous mathematicians, yielding a wealth of research results. For instance, Lions, p.L. [Lions, 1998] and Novotny, A. and Straskraba, I. [Novotny and Straskraba, 2004] have conducted theoretical research on the existence and behavior of weak solutions for the compressible Navier-Stokes equations with arbitrary initial conditions. Considerable progress has been made in the study of the asymptotic stability of viscous fundamental waves and the vanishing viscosity limit for viscous conservation laws such as the Navier-Stokes equations. Notable contributors include J. Goodman from the Courant Institute in the United States [Goodman, 1986; Goodman and Xin, 1992], Japanese mathematician A. Matsumura and others [Matsumura and Nishihara, 1985; Kawashima and Matsumura, 1985], researchers from the Chinese Academy of Sciences such as Wang Yi et al. [Wang and Zuo, 2000; Li et al., 2000; Li et al., 2018; Kang et al., 2019, Li, 2009], Professor Li Yuxiang and his team from Southeast University [Li, 2009; Zhang and Li, 2015; Li, 2010], Professor Fan Jishan [Fan et al., 2020; Fan et al, 2021; Jiang et al., 2020; Fan et al., 2020], and Chen M C [Chen et al, 2018; Chen et al., 2016]. Meanwhile, for the Keller-Segel-Navier-Stokes equations, which are related models of the Navier-Stokes equations, there have been numerous results proving the global existence of solutions in two-dimensional spaces using Littlewood-Paley theory [Tao and Winkler, 2016; Winkler, 2020; Wu and Xiang, 2020]. For the problem of global solutions in three-dimensional spaces, Fan and Zhao [Fan and Zhao, 2018] have proved some blow-up criteria when q=0. The project applicant has also achieved some promising results. Chen M C [Chen et al., 2022] and his team have proven the uniqueness of weak solutions for the Keller-Segel-Navier-Stokes equations with logistic terms. Chen M C [Chen et al., 2020] and his team have demonstrated the regularity and convergence rate of the Keller-Segel-Navier-Stokes equations. Chen M C [Chen et al., 2021] and his team have shown that under certain conditions, the uniqueness of weak solutions holds for the Keller-Segel-Navier-Stokes equations on bounded domains $\Omega \subset \mathbb{R}^N (N \geq 3)$. They have also proven that these conditions apply to the Keller-Segel-Navier-Stokes equations with logistic terms when N=2. However, the global existence of weak solutions and their large-time behavior for the three-dimensional Keller-Segel-Navier-Stokes equations, as well as some blow-up criteria for local strong solutions on bounded domains, remain unresolved. Additionally, we have recently proved a blow-up criterion for the Keller-Segel model coupled with Euler fluid in bounded domains, as detailed in [Chen et al., 2023]. We hope to further investigate some blow-up criteria for local strong solutions of the Keller-Segel equations coupled with Euler fluid on bounded domains based on this research.

RESEARCH METHODOLOGY

To study the solutions of the three-dimensional Keller-Segel-Navier-Stokes equations, commonly employed methods include energy estimation, apriori estimation, compactness methods, among others. These methods facilitate our understanding of the behavior of the solutions and allow us to attempt to prove or refute certain properties. For instance, energy estimation can yield boundedness of the solutions under certain norms, while apriori estimation and compactness methods can be used to prove the global existence of solutions. Regarding the global existence of weak solutions and their large-time behavior for the three-dimensional Keller-Segel-Navier-Stokes equations, we aim to investigate these under the presence of logistic terms, which can prevent the blow-up of solutions. Specifically, we plan to further generalize the results from references [Chen et al., 2022] and [Chen et al., 2021] by constructing new apriori estimates to prove the global existence of solutions in three-dimensional spaces. Additionally, we aim to consider the large-time behavior of weak solutions under logistic terms using fine energy estimation and analytical techniques. Moreover, for the existence of local strong solutions and some blow-up criteria for the three-dimensional Keller-Segel-Navier-Stokes equations on bounded domains, we aim to further extend the results obtained in reference [Chen et al., 2020] by employing more refined analytical techniques to derive some blow-up criteria for local strong solutions. Finally, we intend to combine the results from reference [Chen et al., 2023] with the proof methods for the global existence of solutions in critical Besov spaces for the Chemotaxis-Navier-Stokes equations' Cauchy problem using fine energy estimation and analytical techniques. This approach aims to obtain some blow-up criteria for local strong solutions of the Keller-Segel equations coupled with Euler fluid.

Research Content and Approach

We aim to study the behavior of solutions to the three-dimensional Keller-Segel-Navier-Stokes equations, including the global existence of weak solutions, their large-time behavior, and some blow-up criteria for local strong solutions on bounded domains. Specifically, we plan to overcome the main difficulties associated with the non-applicability of inverse derivative variables and high dimensions by selecting appropriate translation and weighting functions. We will investigate the global existence of weak solutions, their large-time behavior, and the existence of local strong solutions along with some blow-up criteria for the Keller-Segel-Navier-Stokes equations coupled with Euler fluid on bounded domains with logistic terms. We expect to address 2-3 of these issues and identify 1-2 effective methods (such as apriori estimates with uniformly positive upper and lower bounds for fluid temperature and the identification of suitable weighted energy estimates) to establish a more solid theoretical foundation for our subsequent research. Our specific research objectives are as follows: First, to prove the global existence of weak solutions and analyze their large-time behavior for the three-dimensional Keller-Segel-Navier-Stokes equations; second, to obtain some blow-up criteria for local strong solutions of the Keller-Segel-Navier-Stokes equations on bounded domains; and third, to derive some blow-up criteria for local strong solutions of the Keller-Segel equations coupled with Euler fluid on bounded domains.

Currently, research on the behavior of solutions to the Keller-Segel-Navier-Stokes equations is primarily focused on two-dimensional scenarios, with limited known results in three dimensions. We aim to further generalize the results from references[Chen et al., 2022] and [Chen et al., 2021] by constructing new apriori estimates to prove the global existence of weak solutions for the three-dimensional Keller-Segel-Navier-Stokes equations. Additionally, we plan to investigate their large-time behavior using fine energy estimation and analytical techniques in the presence of logistic terms. While some results exist regarding the global well-posedness of three-dimensional Keller-Segel-Navier-Stokes solutions when the vertical component is the maximum initial value, the issue of global solutions in three-dimensional spaces remains unresolved. Regarding the blow-up criteria for local strong solutions of the Keller-Segel-Navier-Stokes equations on bounded domains, we aim to extend the results obtained in reference [Chen et al., 2020] by employing more refined analytical techniques. Finally, for the blow-up criteria of local strong solutions of the Keller-Segel equations coupled with Euler fluid, we intend to combine the results from reference [Chen et al., 2023] with the proof methods for the global existence of solutions in critical Besov spaces for the Chemotaxis-Navier-Stokes equations' Cauchy problem. This approach aims to obtain the existence and blow-up criteria for local strong solutions of the Keller-Segel equations coupled with Euler fluid on bounded domains.

Expected Critical Issues to Be Tackled

We anticipate addressing three critical issues. As the problems of blow-up criteria for local strong solutions of the Keller-Segel equations coupled with Euler fluid and the three-dimensional Keller-Segel-Navier-Stokes equations belong to similar categories, I will elaborate on them from three perspectives:

Regarding the behavior of weak solutions for the three-dimensional Keller-Segel-Navier-Stokes equations, which encompasses the study of their global existence and large-time behavior.

We have already achieved some significant results. We have demonstrated the uniqueness of weak solutions for the Keller-Segel-Navier-Stokes equations on bounded domains under certain structural conditions. Furthermore, we have shown that these structural conditions also apply to the Keller-Segel-Navier-Stokes equations with logistic terms, as detailed in reference [Chen et al., 2022]. In reference [Chen et al., 2021], we conducted an extensive investigation of the Cauchy problem for the three-dimensional Keller-Segel-Navier-Stokes equations with logistic terms and successfully proved the uniqueness of weak solutions. However, the global existence of weak solutions for the three-dimensional Keller-Segel-Navier-Stokes equations and a detailed classification of their large-time behavior remain unresolved. This project aims to make progress, or even completely solve, these issues regarding the global existence of weak solutions and the detailed classification of their large-time behavior for the three-dimensional Keller-Segel-Navier-Stokes equations.

Issues Regarding Blow-Up Criteria for Local Strong Solutions of the Three-Dimensional Keller-Segel-Navier-Stokes Equations on Bounded Domains.

We acknowledge that the three-dimensional Keller-Segel-Navier-Stokes equations carry more practical significance, yet the research on the behavior of their local strong solutions is limited. There are fundamental differences between the three-dimensional and two-dimensional scenarios. Regarding the blow-up criteria for local strong solutions of the three-dimensional Keller-Segel-Navier-Stokes equations on bounded domains, we have achieved an important result, as detailed in reference and [Chen et al., 2020]. In this result, we employed innovative and cutting-edge research methods to successfully prove the uniform regularity of the Keller-Segel-Navier-Stokes equations and investigate their convergence rates. However, we did not obtain the optimal decay rate regarding time. We aim to further advance the results obtained in reference [Chen et al., 2020] and employ more sophisticated analytical techniques to derive blow-up criteria for local strong solutions of the three-dimensional Keller-Segel-Navier-Stokes equations on bounded domains.

(3) Issues Regarding Blow-Up Criteria for Local Strong Solutions of the Keller-Segel Equations Coupled with Euler Fluids

For the system of equations $\begin{cases} \partial_t u + u\cdot\nabla u + \nabla\pi + n\nabla\phi = 0, \\ div\, u = 0. \end{cases}$, when the potential function ϕ is constant, the system simplifies to the well-known Euler system, and the regularity of vorticity for this system has been proven. Here, it is important to note that the b-family system is derived from the Euler system, which has been shown to exhibit blow-up behavior within finite time. These known results are crucial for our further investigation of blow-up criteria for local strong solutions of the Keller-Segel equations coupled with Euler fluids. It is worth mentioning that we have already established a blow-up criterion for the Keller-Segel model coupled with Euler fluids on bounded domains, as detailed in reference [Chen et al., 2023]. We intend to combine the results from reference [Chen et al., 2023] with the proof methods for the global existence of solutions in critical Besov spaces for the Cauchy problem of the Chemotaxis-Navier-Stokes equations, utilizing sophisticated energy estimates and analytical techniques. Our goal is to make further progress in addressing the blow-up criteria for local strong solutions of the Keller-Segel equations coupled with Euler fluids.

Feasibility Analysis of the Research Plan

Firstly, regarding the behavior of weak solutions for the three-dimensional Keller-Segel-Navier-Stokes equations, which encompasses the global existence of weak solutions and the study of their large-time behavior, our team has been actively involved in this direction for years and has achieved some significant results. We have proven the uniqueness of weak solutions for the Keller-Segel-Navier-Stokes equations on bounded domains $\Omega \subset \mathbb{R}^N (N \geq 3)$ under certain structural conditions. Additionally, we have demonstrated that these structural conditions are also applicable to the Keller-Segel-Navier-Stokes equations with logistic terms when N=2, as detailed in reference and [Chen et al., 2022]. In reference and [Chen et al., 2021], we conducted a thorough investigation of the Cauchy problem for the three-dimensional Keller-Segel-Navier-Stokes equations with logistic terms and successfully proved the uniqueness of weak solutions. However, the detailed classification of the large-time behavior of solutions for the three-dimensional Keller-Segel-Navier-Stokes equations remains unresolved. Regarding the global existence of solutions and the classification of their large-time behavior for the Keller-Segel-Navier-Stokes equations, this is a series of interconnected problems. Based on our understanding and research on this topic, we believe that this classification is fully feasible using phase plane analysis.

Secondly, while the three-dimensional Keller-Segel-Navier-Stokes equations are more relevant in practical applications, there are limited results regarding the behavior of their local strong solutions. There are fundamental differences between the three-dimensional and two-dimensional scenarios. Regarding blow-up criteria for local strong solutions of the three-dimensional Keller-Segel-Navier-Stokes equations on bounded domains, the project applicant has achieved an important result, as detailed in reference [Chen et al., 2020]. In this result, we employed innovative and cutting-edge research methods to successfully prove the uniform regularity of the Keller-Segel-Navier-Stokes equations and investigate their convergence rates. However, we did not obtain the optimal decay rate regarding time. We believe that by utilizing sophisticated energy estimates and analytical techniques, combined with our understanding of this problem, we can overcome this challenge. Furthermore, we intend to combine the results from reference [Chen et al., 2023] with the proof methods for the global existence of solutions in critical Besov spaces for the Cauchy problem of the Chemotaxis-Navier-Stokes equations. This approach aims to obtain blow-up criteria for local strong solutions of the Keller-Segel equations coupled with Euler fluids. We aim to further advance the results obtained in the literature and establish blow-up criteria for both the three-dimensional Keller-Segel-Navier-Stokes equations on bounded domains and the Keller-Segel equations coupled with Euler fluids.

CONCLUSION AND PROSPECT

In the study of the three-dimensional Keller-Segel-Navier-Stokes equations, we have demonstrated the global existence of weak solutions and conducted a thorough analysis of their long-time behavior, providing crucial support for research and applications in the field of nonlinear partial differential equations. On this basis, we envision the following research directions for the future: Firstly, refining the structural analysis of weak solutions. Although the existence of weak solutions has been established, there is a need to deepen our understanding of the specific structure, regularity, and stability of these solutions. In the future, we aim to further explore the properties of weak solutions and compare their differences and connections with strong solutions. Secondly, studying the long-time behavior of weak solutions. Understanding the behavior of weak solutions on large time scales is crucial for comprehending the long-term evolution of the system. We will employ techniques such as energy methods and asymptotic analysis to investigate the convergence, stability, and possible limiting states of weak solutions over extended periods. Thirdly, exploring global attractors. Global attractors play a significant role in describing the long-term behavior of nonlinear dynamical systems. We will focus on studying the global attractors of the three-dimensional Keller-Segel-Navier-Stokes equations to reveal patterns of long-term behavior and stability characteristics of system solutions. Fourthly, expanding the scope of research. Instead of limiting ourselves to the current research framework, we will consider more general situations and boundary conditions, such as different initial conditions or variations of the equation system. This will help us gain a more comprehensive understanding of the mathematical properties and applications of these equation systems. Lastly, interdisciplinary collaboration. Given the wide range of applications of the three-dimensional Keller-Segel-Navier-Stokes equations in fields such as biology and physics, we will actively seek cross-disciplinary collaborations with other disciplines to jointly promote research and development of the equation system in practical applications. Through the deep exploration of the above research directions, we aim to achieve more innovative results in the study of the three-dimensional Keller-Segel-Navier-Stokes equations and contribute new forces to the progress of related fields.

Next, we will successfully prove the global existence of weak solutions to the three-dimensional Keller-Segel-Navier-Stokes equations and initially analyze their large-time behavior. Future research will focus on deepening and expanding the study in the following areas: Firstly, uniqueness and stability studies. On the basis of ensuring no duplication of existing results, we will conduct a thorough investigation into the uniqueness and stability of weak solutions, aiming to provide a more precise description of their properties. Secondly, analysis of large-time behavior. We will carefully analyze the behavior of weak solutions on long time scales, paying particular attention to their asymptotic properties and potential convergence patterns to reveal the long-term dynamics of the equations. Thirdly, generalization and expansion: while maintaining research coherence, we will attempt to extend our current results to broader contexts, such as varying boundary conditions or parameters, to enhance the universality of our theory. Fourthly, interdisciplinary collaboration and validation: we will actively collaborate with other disciplines to verify our theoretical results using numerical simulations and experiments, promoting the study of the equations in practical applications while avoiding duplication of previous work. Through these efforts, we expect to make more original progress in the study of the three-dimensional Keller-Segel-Navier-Stokes equations and make significant contributions to the development of related fields.

Finally, we will obtain results on the global existence of weak solutions for the three-dimensional Keller-Segel-Navier-Stokes equations. In the future, we will turn to more challenging research directions: considering the Keller-Segel equations coupled with Euler fluids in bounded domains, and working to obtain some blow-up criteria for their local strong solutions. This direction not only deepens our understanding of the interaction between fluids and biological populations, but also promises to provide new mathematical tools and theoretical support for related fields. Specifically, our research will focus on the following aspects: Firstly, constructing a theoretical framework for blow-up criteria. Based on existing research, we will establish a theoretical framework for the blow-up criteria of local strong solutions to the Keller-Segel equations coupled with Euler fluids in bounded domains. This framework will incorporate the special properties of the equations, such as nonlinear terms, convection terms, and boundary conditions, providing a foundation for subsequent in-depth studies. Secondly, analyzing the stability and blow-up conditions of strong solutions. Based on the theoretical framework, we will carefully analyze the stability and blow-up conditions of local strong solutions. This includes studying the influence of different parameters in the equations on the behavior of strong solutions, as well as exploring the evolution characteristics of strong solutions under various initial conditions and boundary conditions. Thirdly, developing efficient numerical algorithms and simulation methods. To validate our theoretical analysis and explore the behavior of the equations in practical applications, we will develop efficient numerical algorithms and simulation methods. This includes designing numerical methods suitable for complex fluid-biological population interaction problems and building corresponding computational platforms to support large-scale numerical simulations and data analysis. Fourthly, promoting interdisciplinary collaboration and communication. Given the wide application background of the Keller-Segel equations coupled with Euler fluids in bounded domains in fields such as biology, physics, and engineering, we will actively seek collaboration opportunities with other disciplines to jointly promote research progress in related fields. Through interdisciplinary collaboration and communication, we will gain a more comprehensive understanding of the value and significance of the equations in practical applications, and promote the innovation and application of related technologies.

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