Coordination Orders of Joint Procurement Through Quantity Discounts

Hong-Wei Liu[1]

[1] *School of Information, Beijing Wuzi University, Beijing 101149, China*

Abstract*:* Coordination mechanism under the well-known one supplier multiple retailer model with quantity discount schedule where constant customer demand occurs at each retailer over an infinite time horizon is investigated. Compared to fixed wholesale price policy, quantity discounts policy eliminates supply chain cost to some extend and among supplier and retailers are able to share system cost saving benefits. If retailers' order quantity and supplier's order processing cost are more reduced, channel coordination effectiveness can be achieved at a higher level and system cost can be further eliminated. Furthermore, if retailers employee joint replenishment strategy, channel coordination effectiveness can be achieved at a higher level and system cost can be further eliminated. The numerical experiment result shows that the quantitative discounts provided by the supplier can achieve channel coordination depends intimately upon the replenishment model applied by the retailers and these could be advantageous in many environments.

Keywords Supply chain management; Channel coordination; Joint procurement; Quantity discounts; EOQ

INTRODUCTION

In the business world of today, such as fashion apparel, popular toys, etc., the issue of long lead times and short product life cycles forces retailers to make procurement decisions while there is still a great deal of uncertainty regarding demand. To make these newsvendor procurement decisions, the retailers try to maximize their own profits by balancing the potential costs associated with unsatisfied demand and excess stock. Unfortunately, there exist a variety of reasons why the quantities the retailers choose fail to maximize the profits the supply chain as a whole.

In this paper, we address coordination mechanism design under the well-known one supplier multiple retailer model where constant customer demand occurs at each retailer over an infinite time horizon and quantity discount schedule is employed. Quantity discounts are widely used by the seller with the objective of inducing the buyer to order larger quantities in order to reduce their total transaction costs associated with ordering, shipment and inventorying. Although much previous research has considered coordination issues in supply chains, most of these papers suggesting pricing strategies to coordinate supply chain participants have considered either a single price discount or a discount based on order quantities. In contrast, we suggest a economic order quantity (EOQ) policy offered to all retailers. The classical EOQ model is a cost-minimization inventory model with a constant demand rate. Furthermore, we feel that the policies described in this paper are feasible and cost effective under certain conditions. The objectives of our study are threefold. First, we develop an efficient methodology for finding the optimal discount price and describe conditions which must exist for a manufacturer to consider offering such a discount. Second, we investigate the magnitude of the savings which might accrue to a manufacturer who offers such a discount. Finally, we wish to identify the conditions when such a scheme would be beneficial to all supply chain participants. We believe that our findings offer significant managerial implications in the area of channel management and supply chain coordination when these systems are centralized.

The remainder of the paper is organized as follows. Following a description of related research, we describe and analyze three replenishment strategies in section 2. In the third section, we present results from a numerical experiment and discuss the managerial implications. In the final section, we summarize the implications of our proposed policy and present several extensions to consider in future research.

RELATED WORK

In recent years, a large amount of attention has been devoted to studying channel coordination in newsvendor environments. Jeuland [Jeuland et al., 1983] considered the issue of channel coordination from a marketing perspective and discussed several mechanisms that could improve channel coordination, while Dolan [Dolan, 1987] classified several different conditions under which quantity discount scheme for heterogeneous buyer types could lead to higher profits as a result of improved channel coordination. Monahan[Monahan,1984] formulated the transaction between the seller and the buyer (see also [Dada et al., 1987], [Rosenblatt et al.,1985]), and proposed a

method for determining an optimal all-unit quantity discount policy with a fixed demand. Lee and Rosenblatt[Lee et al., 1984] generalized Monahan's model to obtain the "exact" discount rate offered by the seller, and to relax the implicit assumption of a lot-for-lot policy adopted by the seller. Crowther [Crowther, 1967] and Lal [Lal et al., 1984] considered both buyer and seller costs to justify the implementation of a quantity discounts. Dada [Dada et al., 1987] extended the work by Lal [Lal et al., 1984] and suggested optimal pricing policies as well as a mechanism for allocating the cost savings between the seller and the buyer. Parlar [Parlar et al.,1995] proposed a model using the pricing decision of a supplier and the subsequent ordering decisions of homogeneous buyers to analyze the quantity discount problem as a perfect information game. For more work: see also Sarmah [Sarmah et al.,2006]. Their work was extended by Wang [Wang et al., 2000] to the case with heterogeneous retailers. Weng [Weng, 1995] analyzed the effects of joint decision policies on channel coordination in a distribution system which consists of a single seller and a group of homogeneous buyers when demand is price sensitive and operating costs are functions of order quantities. A good review of the operations literature on quantity discounts can be found in Viswanthan [Viswanthan et al., 2003], who classify the literature according to whether there is one or multiple buyers and whether there is price sensitive demand. These models assumed that both the seller's and the buyer's inventory policies can be described by classical EOQ models. It is one of the most successful models in all the inventory theories due to its simplicity and easiness.

Since our research involves a quantity discounts, it is worth pointing out that, nearly all of the work on quantity discounts has focused on all-unit discounts. Although both Kim [Kim et al., 1988] and Weng [Weng, 1995] consider incremental discounts, neither one examines the question of whether a supplier should prefer one to the other. In fact, Weng [Weng, 1995] shows that in an EOQ context, either an all-unit or incremental discount can be used to achieve channel coordination, and that the supplier would be indifferent between the all-unit and incremental discount. Our work fills this result by illustrating that a seller will prefer the all-unit discount in a newsvendor context.

MATERIALS AND METHODS

One supplier multiple retailer model here is a multiple retailer situation where each retailer is faced with basic EOQ problem and where the cost of each retailer is the sum of a first setup cost, a second holdings cost and a third stock cost. We will discuss situation under the supplier different polices. For simplicity of analysis, we denote N the finite set of retailers and 0 denote the supplier. The parameters

associated to every $i \in N \cup \{0\}$ in one of those systems are:

· $R_i > 0$, the fixed setup cost per order;

· $h_i > 0$, the holding cost per item and per time unit;

· $D_i > 0$, the deterministic demand per time unit,

such that $D_0 = \sum_{i=1}^{n} D_i$.

Each retailer $i \in N$ has to meet the demand in time. To attain this, i keeps stock in hand by placing order size $Q_i > 0$. We first assume that the supplier sell the stocks with fixed wholesale price w and retailers make decision to order independently. It is well known that the optimal size of the order and the minimum cost for retailer i are

$$Q_i = \sqrt{\frac{2D_i R_i}{h_i}},$$

and

$$\prod_i (Q_i, w) = w D_i + R_i \frac{D_i}{Q_i} + h_i \frac{Q_i}{2}.$$

Since the heterogeneous property of retailers, for simplicity, we let $Q_i > Q_{i-1} (i > 1)$. Define the vector $Q = (Q_1, Q_2, \cdots, Q_n)^T$, thus minimum cost for the supplier is

$$\prod_0 (Q, w) = R_0 \sum_{i=1}^{n} \frac{D_i}{Q_i} + h_0 \sum_{i=1}^{n} \frac{Q_i}{2} - w \sum_{i=1}^{n} D_i,$$

and minimum cost for the system is

$$\prod (Q) = \sum_{i=1}^{n} \frac{D_i}{Q_i} (R_0 + R_i) + \sum_{i=1}^{n} \frac{Q_i}{2} (h_0 + h_i).$$

Coordination Mechanism under Quantity Discounts with Decentralized Order

For retailer i, if the supplier take the w as the benchmark price when optimal order quantity Q_i, he must design optimal quantity discount scheme with discount price w_i^d to make retailer i's incremental order form Q_i to Q_i^d. The retailer still place order individual, and the minimum cost for retailer i

$$\prod_i (Q_i^d, w_i^d) = w_i^d D_i + R_i \frac{D_i}{Q_i^d} + h_i \frac{Q_i^d}{2}.$$

Denote $Q^d = (Q_1^d, ..., Q_n^d)^T$ and $w^d = (w_1^d, ..., w_n^d)^T$, thus minimum cost for the supplier is

$$\prod_0 (Q^d, w^d) = R_0 \sum_{i=1}^{n} \frac{D_i}{Q_i^d} + h_0 \sum_{i=1}^{n} \frac{Q_i^d}{2} - \sum_{i=1}^{n} w_i^d D_i$$

and minimum cost for the system is

$$\prod (Q^d) = \sum_{i=1}^{n} \frac{D_i}{Q_i^d} (R_0 + R_i) + \sum_{i=1}^{n} \frac{Q_i^d}{2} (h_0 + h_i).$$

Take the optimal systematic cost (6) as principle, the optimal discount quantity should be

$$Q_i^d = \sqrt{\frac{2D_i(R_i + R_0)}{h_i + h_0}}, \quad i = 1, 2, \ldots, n.$$

Since the heterogeneous property of retailers, for simplicity, we let $Q_i^d > Q_{i-1}^d (i > 1)$.

In general, the order under effective quantity discounts is greater than the one under fixed price [Rosenblatt *et al.*,1985], and the setup cost of the supplier is greater than that of retailers [Dada *et al.*, 1987]. Without loss of generality, we let $Q_i^d > Q_i$. Thus the supplier will give a decreasing wholesale price strategy ($w_i^d \le w_{i-1}^d \le w$) to encourage the retailer enhancing the order level under retailers' participant and incentive compatibility constraint.

· Participant Constraint of Retailers:

The retailer i will increase his order from Q_i to Q_i^d if $w_i^d \le w_{i,\max}^d$, where $w_{i,\max}^d = \frac{B_i^d}{D_i}$ and

$$B_i^d = wD_i + R_i(\frac{D_i}{Q_i} - \frac{D_i}{Q_i^d}) - h_i\frac{Q_i^d - Q_i}{2}.$$

Proof: when $\prod_i(Q_i, w) \ge \prod_i(Q_i^d, w_i^d)$, the retailer will increase his order. From (1) and (4), we have

$$wD_i + R_i\frac{D_i}{Q_i} + h_i\frac{Q_i}{2} \ge w_i^dD_i + R_i\frac{D_i}{Q_i^d} + h_i\frac{Q_i^d}{2}.$$

After computing, the result gets.

The discount price should encourage retailer to enhance the order from Q_i to Q_i^d but not to $Q_j^d (j \ne i)$. We define discount difference series as

$$\Delta_{i-1}^d = w_{i-1}^d - w_i^d, \quad i = 2, 3, \ldots, n.$$

· Incentive Compatibility Constraint of retailers:
If $\Delta_{i-1,d}^d \le \Delta_{i-1}^d \le \Delta_{i-1,u}^d$, effective order of retailers should not deviate from their optimal order level, where

$$\Delta_{i-1,d}^d = \frac{1}{D_i}\left[h_i\frac{Q_i^d - Q_{i-1}^d}{2} - R_i(\frac{D_i}{Q_{i-1}^d} - \frac{D_i}{Q_i^d})\right]$$

and

$$\Delta_{i-1,u}^d = \frac{1}{D_{i-1}}\left[h_{i-1}\frac{Q_i^d - Q_{i-1}^d}{2} - R_{i-1}(\frac{D_{i-1}}{Q_{i-1}^d} - \frac{D_{i-1}}{Q_i^d})\right].$$

Proof: From the constraint $\prod_i(Q_i^d, w_i^d) \le \prod_i(Q_{i-1}^d, w_{i-1}^d)$ and $\prod_{i-1}(Q_{i-1}^d, w_{i-1}^d) \le \prod_{i-1}(Q_i^d, w_i^d)$, we can get the result.

· Participant Constraint of Supplier:
Retailer increasing order will improve the benefit of the supplier and the discount price given by supplier satisfy $w_{i,\min}^d \le w_i^d$, where

$$w_{i,\min}^d = h^d - \frac{\Phi_i^d}{\sum_{i=1}^{n} D_i}$$

and

$$h^d = \frac{w\sum_{i=1}^{n} D_i - R_0\sum_{i=1}^{n}\left(\frac{D_i}{Q_i} - \frac{D_i}{Q_i^d}\right) + h_0\sum_{i=1}^{n}\frac{Q_i^d - Q_i}{2}}{\sum_{i=1}^{n} D_i}$$

and $\Phi_i^d = P_d^T G_i I$ where

$$P_d = (\Delta_1^d, \cdots, \Delta_{i-1}^d, \Delta_i^d, \Delta_{i+1}^d, \cdots, \Delta_{n-1}^d)^T,$$

$$I = (1, \cdots, 1)_{(n-1)\times(n-1)}^T, \quad G_i = \begin{bmatrix} A_i & 0 \\ 0 & -B_i \end{bmatrix},$$

$$A_i = \begin{bmatrix} D_1 & & & \\ D_1 & D_2 & & \\ \vdots & \vdots & \ddots & \\ D_1 & D_2 & \cdots & D_{i-1} \end{bmatrix}, B_i = \begin{bmatrix} D_{i+1} & D_{i+2} & \cdots & D_n \\ & D_{i+2} & \cdots & D_n \\ & & \ddots & \vdots \\ & & & D_n \end{bmatrix}.$$

Coordination Mechanism under Quantity Discounts with Centralized Order

Under the basic EOQ policy, the frequency of order by retailers are different. If all retailers wish to change order plan and place order within the same time window, the coordinated cost will decrease in a great deal [Viswanathan, *et al.*, 2001], [Mishra, 2004], [Klastorin, *et al.*, 2002]. Considering all retailers place joint procurement in coordination, and denoted $\frac{Q_1}{D_1} = \frac{Q_i}{D_i}(i \ne 1)$. The system cost structure are as following:

The minimum cost for retailer i

$$\prod_i(Q_i^c, w_i^c) = w_i^cD_i + R_i\frac{D_i}{Q_i^c} + h_i\frac{Q_i^c}{2}.$$

Denote $Q^c = (Q_1^c, \ldots, Q_n^c)^T$ and $w^c = (w_1^c, \ldots, w_n^c)^T$, thus minimum cost for the supplier is

$$\prod_0(Q^c, w^c) = R_0\sum_{i=1}^{n}\frac{D_i}{Q_i^c} + h_0\sum_{i=1}^{n}\frac{Q_i^c}{2} - \sum_{i=1}^{n} w_i^cD_i$$

and minimum cost for the system is

$$\prod(Q^c) = \sum_{i=1}^{n}\frac{D_i}{Q_i^c}(R_0 + R_i) + \sum_{i=1}^{n}\frac{Q_i^c}{2}(h_0 + h_i).$$

Take the optimal systematic cost (13) as principle, the optimal discount quantity should be

$$Q_i^c = D_i\sqrt{\frac{2(\sum_{i=1}^{n} R_i + R_0)}{\sum_{i=1}^{n} D_i(h_i + h_0)}}, \quad i = 1, 2, \ldots, n.$$

Similar to the situation of decentralize quantity discount, we assume that

$$Q_i^c > Q_{i-1}^c, i = 2, \cdots, n, \quad Q_i^c > Q_i, i = 1, \cdots, n.$$

Similar discussion to the situation of decentralize quantity discount, we get the following properties:
· Participant Constraint of Retailers:

The retailer i will increase his order from Q_i to

Q_i^c if $w_i^c \leq w_{i,\max}^c$ where $w_{i,\max}^c = \dfrac{B_i^c}{D_i}$ and

$$B_i^c = wD_i + R_i\left(\frac{D_i}{Q_i} - \frac{D_1}{Q_1^c}\right) - h_i\frac{Q_i^c - Q_i}{2}.$$

The discount price should encourage retailer to enhance the order from Q_i to Q_i^c but not to $Q_j^c (j \neq i)$. We define discount difference series as

$$\Delta_{i-1}^c = w_{i-1}^c - w_i^c, \quad i = 2, 3, \ldots, n.$$

· Incentive Compatibility Constraint of retailers :

If $\Delta_{i-1,d}^c \leq \Delta_{i-1}^c \leq \Delta_{i-1,u}^c$, effective order of retailers should not deviate from their optimal order level, where

$$\Delta_{i-1,d}^c = \frac{1}{D_i}\left[h_i\frac{Q_i^c - Q_{i-1}^c}{2} - R_i\frac{D_1}{Q_1^c}(\frac{Q_c^c}{Q_{i-1}^c} - 1)\right]$$

and

$$\Delta_{i-1,u}^c = \frac{1}{D_{i-1}}\left[h_{i-1}\frac{Q_i^c - Q_{i-1}^c}{2} - R_{i-1}\frac{D_1}{Q_1^c}(1 - \frac{Q_{i-1}^c}{Q_i^c})\right].$$

· Participant Constraint of Supplier :

Retailer increasing order will improve the benefit of the supplier and the discount price given by supplier satisfy $w_{i,\min}^c \leq w_i^c$, where

$$w_{i,\min}^c = h^c - \frac{\Phi_i^c}{\sum\limits_{i=1}^{n} D_i}$$

and

$$h^c = \frac{w\sum\limits_{i=1}^{n} D_i - R_0\left(\sum\limits_{i=1}^{n}\frac{D_i}{Q_i} - \frac{D_1}{Q_1^c}\right) + h_0\sum\limits_{i=1}^{n}\frac{Q_i^c - Q_i}{2}}{\sum\limits_{i=1}^{n} D_i}.$$

and $\Phi_i^c = P_c^T G_i I$ where

$$P_c = (\Delta_1^c, \cdots, \Delta_{i-1}^c, \Delta_i^c, \Delta_{i+1}^c, \cdots, \Delta_{n-1}^c)^T.$$

RESULTS AND DISCUSSION

We consider an example in order to assess the benefit of quantity discounts to the supplier as well as to retailers and to explore the optimal discount policies with respect to the model parameters. To

estimate the benefit of quantity discounts to the supplier we first must analyze his optimal pricing policy without quantity discounts, i.e., when a single common price is offered to all retailer types. We refer to this case as the fixed price problem. For the numerical experiment, we consider a situation with $n = 2$ retailers and fixed price is $w = 10$. The market sizes are $D_1 = 1000$, $D_2 = 1500$ and the fixed setup cost per order are $R_0 = 200, R_1 = 50, R_2 = 75$, and the holding cost per item are $L_0 = 0.5, L_1 = 2.0, L_2 = 1.5$. Table 1 shows the optimal quantity discount policies in decentralized and centralized situations and supplier profits as well as the comparison to the single price benchmark case. From these results it can be observed that, the supplier's profit as well as the retailers' increases with quantity discounts. This is consistent with the fact that, in any news vendor setting, the supplier's expected profit is higher under quantity discount than under a single price benchmark case as well as the retailers. It is interesting to examine the magnitude of the difference which is shown in the far right column of Table 1.

CONCLUSION

In this article, coordination mechanism under the well-known one supplier multiple retailers model with quantity discount schedule in different situations where constant customer demand occurs at each retailer over an infinite time horizon is investigated. We have paid main attention to discuss the benefit of quantity discounts to the supplier as well as to retailers in supply chain. Compared to fixed wholesale price policy, quantity discounts policy eliminates supply chain cost to some extend and supplier and retailers are able to share system cost saving benefits in decentralized situation. Furthermore, if retailers employee joint replenishment strategy, channel coordination effectiveness can be achieved at a higher level and system cost can be further eliminated. In result section, a numerical experiment is discussed and the result shows that the quantity discounts provided by the supplier can achieve channel coordination depends intimately upon the replenishment model applied by the retailers in different situations and these could be advantageous in many other environments.

Table 1:The results of comparing cost saving in different situations.

facilities	with Fix Price				with Decentralized Order				with Centralized Order			
	D_i	R_i	H_i	Q_i	w_i^d	Q_i^d	$w_{i,max}^d$	*saving*	w_i^c	Q_i^c	$w_{i,max}^c$	*saving*
Retailer1	1000	50	2.0	223.6	9.856	447.2	9.888	32.2	8.840	343.8	9.958	102
Retailer2	1500	75	1.5	387.3	9.754	642.3	9.949	293.1	9.718	515.7	9.984	345
Supplier		200	0.5					122.1				512.1
System								447.4				959.1

REFERENCES

Cachon G., 1999, "Managing Supply Chain Variability with Scheduled Ordering Policies", Management Science., vol.45, pp 843-856.

Chen F., Federgruen A., Zheng Y., 2001, "Coordination mechanisms for a distribution system with one supplier and multiple retailers", Management Science, vol.47,No.5, pp 693-708.

Crowther J.F., 1967, "Rationale for quantity discounts", Harvard Business Review, vol.42, pp 121-127.

Dada M., Srikanth K.N., 1987, "Pricing policies for quantity discounts", Manangement Science, vol.33,No.10, pp 1247-1252.

Dolan R.J., 1987, "Quantity discounts: Managerial issues and research opportunities", Marketing Science, vol.6, pp 1-22.

Jeuland A., Shugan S., 1983, "Managing channel profits", Marketing Science, vol.2, pp 239-272.

Kim K., Hwang H., 1988, "An incremental discount pricing schedule with multiple customers and single price break", European Journal of Operational Research, vol. 35, pp 71-79.

Klastorin T.D., Moinzadeh K., Son J., 2002, "Coordinating orders in supply chains through price discounts", IIE Transactions, vol.34,No.8, pp 679-689.

Lal R., Staelin R., 1984, "An approach for developing an optimal discount pricing policy", Management Science, vol.30,No.12, pp 1524-1539.

Lee H. L., Rosenblatt M. J., 1986, "A generalized quantity discount pricing model to increase vendor's profit," Management Sci., vol. 32, No. 9, pp 1177–1185.

Mishra A.K., 2004, "Selective discount for supplier-buyer coordination using common replenishment epochs", European Journal of Operational Research, vol.153,No.3, pp 751-756.

Monahan J.P., 1984 "A quantity discount pricing model to increase vendor's profit" ,Management Sci., vol. 30, No. 6, pp 720–726.

Parlar M., Wang Q., 1994, "Discounting Decisions in a Supplier-Buyer Relationship with a Linear Buyer's Demand", IIE Transactions, vol.26,No.2, pp 34-41.

Rosenblatt M.J., Lee H.L., 1985, "Improving profitability with quantity discounts under fixed demand",IIE Transactions, vol.17,No.4, pp 388-395.

Sarmah S.P., Acharya D., Goyal S.K., 2006, "Buyer vendor coordination models in supply chain management," European Journal of Operational Research, vol. 175, No. 1, pp. 1–15.

Wang Q., Wu Z., 2000, "Improving a Supplier's Quantity Discount Gain from Many Different Buyers", IIE Transactions,vol. 32, pp 1071-1079.

Weng Z.K., 1995, "Channel coordination and quantity discounts", Management Science, vol.41,No.9, pp 1509-1522.

Viswanathan S., Piplani R., 2001, "Coordinating supply chain inventories through common replenishment epochs", European Journal of Operational Research, vol.129,No.2, pp 277-286.