

# Application of Combination Forecasting Model in the Patrol Sales Forecast

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**Abstract:** After analyzing the inventory and condition sales of patrol station, combination forecasting model for the patrol sales is established by combining the gray system model and the BP-neural model based on time series. During the combination, nonlinear programming problem is applied to minimize the sum of squares of the average predict error, whose variables are coefficients of the combination forecasting model. Then, an example is given to compare the accuracy of the gray system model, the BP-neural model and the combination forecasting model. It proves that the combination forecasting model is superior.

**Keywords:** Gray System; BP-neural network; combination forecasting model; patrol sales

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## INTRODUCTION

As one of the most important strategic resources, oil resources affecting the country's security, social and political stability. The supply of the oil is the basis for economic and social development of each country. At the same time, with the development of the automotive industry, sales of patrol station are increasing. Meeting the market demand for patrol is the primary target of patrol stations. What has become an important premise of patrol supplication is to forecast the sales due to the requirement that out of stock is not allowed. Therefore, accurately forecasting demand of petrol is of paramount importance as shortage will lead to highly undesirable consequences.

Theories and methods about sales forecasting has drawn many experts' attention. Some of the achievements has been applied to the prediction of the petrol demand. Huazhi Liu forecasted the demand of the petrol market in Sichuan province of China by using the method of regression analysis. [Liu et al., 2004] In a similar endeavor, Ge Jing also forecasted it, with the consumption elasticity coefficient method which he indicated is better for long horizon prediction. The gray system, however is more reliable for short term forecast [Jing et al., 2007]. Gray system was widely been and has been applied by Guohua He to forecast regional logistic demand [He et al., 2008]. Qingyuan Li established a forecast model by combining dynamic management model with safety stock, demand forecasting model, vehicle loading model and vehicle routing model [Li et al., 2008] Jianguo Zhang who combined the gray system and BP-neural model [Zhang et al., 2008]. Qiaoyun Wei forecasted the petrol demand by the exponential smoothing method [Wei et al., 2008] while system

dynamics method was used by Lei Yuan [Lei et al., 2010]. Siyu Shi predicted the demand of automobile by building a linear combination forecasting model based on improved gray theory-BP neural network-support vector machine [Shi et al., 2012]. The improved grey markov chain model is established using the lide transition probability matrix by Ling Zhao to forecast the traffic accident death rate [Zhao et al., 2013]. Zhenpeng Jia proposed a kind of support vector machine method [Jia et al., 2014] and Zhigang Sheng compared the single, second, cubic of exponential smoothing and Holter-Winter's exponential smoothing method for the demand prediction of diesel oil [Sheng et al., 2014].

In addition to historical data that can be used as a reference for forecasting, the sales of patrol is additionally affected by weather, traffic conditions, day of the week, whether the limit line and equipment maintenance. The factor impacting the sales of the patrol should be selected with prudence. Due to the difficulty to get the accurate information of the weather and traffic conditions, we take the time series method considering the certain regularity in time of working days, price change and limit line.

Time series method is a kind of traditional way for sales prediction by analyzing the basic rule of the development of the observed value in a time sequence within the same period. Generally, time series forecasting method commonly use the moving average, exponential smoothing method, the grey system theory and markov prediction, etc. It is concluded that the prediction result, however, by the forecasting model above signal used, is difficult to achieve required prediction accuracy. Therefore, it is effective to combine multiple forecasting models. In the process of establishing the model, we make a

hypothesis that maintenance status of the petrol station does not happened.

The remainder of this paper is organized as follows. We established the combination forecasting model after building the gray system model and the BP-neural model based the time series in section 2. An example is given in section 3. Summary of the research is in section 4.

**FORECASTING MODEL**

**Gray system model**

The grey system theory is a method to establish the dynamic model of the differential equation by using the discrete data column after defining the grey derivative and grey differential equation based on the concept of related space and smooth discrete function. [Shi et al., 2010] This kind of Model is called Grey Model (GM), for its attributes of approximate, non-unique. And GM (1, 1) model is one of the most commonly used models. If we build the model based on the data column  $x^{(0)}$ , the procedure is as follows:

1) After the original data accumulated in order to weaken the volatility and randomness of random sequence, the new data sequence we get is like this:

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n))$$

In the function,  $x^{(1)}(t)$  represent the accumulation of the data from 1 to t.

$$x^{(1)}(t) = \sum_{k=1}^t x^{(0)}(k), \quad t = 1, 2, \dots, n$$

$$\text{or } x^{(1)}(t+1) = \sum_{k=1}^{t+1} x^{(0)}(k), \quad t = 1, 2, \dots, n$$

2) Build the first order linear differential equation.

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = u$$

In the function,  $a$  and  $u$ , which called development coefficient and grey action, are the undetermined coefficients. The matrix

consisting  $a$  and  $u$  can be written as  $\hat{a} = \begin{pmatrix} a \\ u \end{pmatrix}$ . after

finding out the value of  $a$  and  $u$ , the predictive value of  $x^{(0)}$  can be known from the value of  $x^{(1)}(t)$ .

3) Calculate the mean value of the accumulated data to generate

$$B = \begin{bmatrix} 0.5(x^{(1)}(1) + x^{(1)}(2)) \\ 0.5(x^{(1)}(2) + x^{(1)}(3)) \\ \vdots \\ 0.5(x^{(1)}(n-1) + x^{(1)}(n)) \end{bmatrix},$$

and  $Y_n = (x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(n))^T$ .

4) Solve the grey coefficient  $\hat{a}$ , and

$$\hat{a} = \begin{pmatrix} a \\ u \end{pmatrix} = (B^T B)^{-1} B^T Y_n$$

5) After putting the grey coefficient  $\hat{a}$  into and solve the equation  $\frac{dx^{(1)}}{dt} + ax^{(1)} = u$ , we get the function:

$$x^{(1)}(t+1) = (x^{(0)}(1) - \frac{u}{a})e^{-at} + \frac{u}{a}$$

Because  $\hat{a}$  is the Approximate value through the least square method,  $x^{(1)}(t+1)$  is an approximate expression.

6) Discrete and Strive for the difference between  $x^{(1)}(t+1)$  and  $x^{(1)}(t)$  to reduce the original sequence  $x^{(0)}$  as follow:

$$x^{(0)}(t+1) = x^{(1)}(t+1) - x^{(1)}(t)$$

**BP-neural model**

Back Propagation neural network (BP-neural network), putted forward by Rumelhart and McClland in 1986, is currently one of the most widely used neural network model. BP-neural network can learn and store numerous input - output model mapping relation, without introduce the equation which describe the relationship. Its learning rule is to use the steepest descent method to minimize the squares of error sum of the network, by constantly adjusting the network weights and threshold though back propagation. [Shi et al., 2012] Topological structures of BP-neural network model include input layer the hidden layer and output layer, which are shown in figure 1.

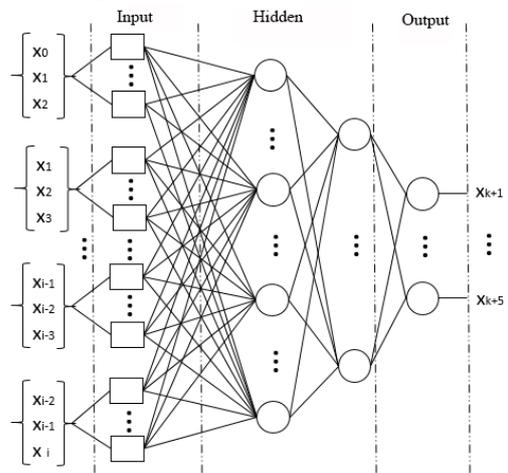


Figure 1. Topological structures of BP-neural network

BP-neural network, modeled based the internal relations between the data, has good self-organizing and adaptability, strong learning ability and anti-

interference ability. It automatically extracts knowledge from data, overcomes many limitations of traditional quantitative prediction method and facing difficulties and at the same time, avoids the influence of some human factors. Between the input layer and output layer of the BP neural network, variables are determined by the weights between the different layers. Regulation of the weights is mainly through the study of signal. So, the more they learn, the more intelligent the network will be; in addition, the number of hidden layers and the accuracy of the network are positively related.

To forecast sales by the BP neural network, the procedure should be divided into several steps as follows:

Step1: Select the appropriate training sample data. In the time series analysis, the historical data has certain effect on the sales volume. Consider sales of the petrol in a certain period, the same period of the previous year should be taken into account.

Step2: Pre-processing of sample data. Normalized the sales sample data to avoid null result for output caused by too much sample data. In order to eliminate noise, result the normalized data in [0, 1].

Step3: Construct the training sample. We put two days' data to be predicted and the historical data of the same period the year before as training sample of the network model.

Step4: Test the network model.

**Combination forecasting model**

Combination forecast model is built by combining two or more single forecasting model based on the form of the appropriate weighted. Due to the defect of these single forecasting model, prediction results are often ineffective, while combination forecasting model performs better for its ability to consider the information synthetically[Shi et al., 2010]. Based on the functional relationship between combination forecast model and single forecasting model, combination forecast model can be divided into linear combination forecasting model and nonlinear combination forecasting model. Here we use linear combination forecasting model.

We assume that there are  $n$  kind of forecast methods,  $x_t$  is the real value in the time unit  $t$ ,  $x_{it}$  and  $e_{it}$  is the forecasting value and the error in the time unit  $t$  by the method  $i$ . And  $e_{it} = x_t - x_{it}$ , ( $i = 1, 2, \dots, n; t = 1, 2, \dots, N$ ), where  $l_i$  represent the weighted of method  $i$  and

$$\sum_{i=1}^n l_i = 1, l_i \geq 0, i = 1, 2, \dots, n.$$

Squares sum of error may reflect the size of the forecasting error. Based on this, the mathematical model established is as follows:

$$\min f = \sum_{t=1}^N [x_t - \sum_{i=1}^n l_i x_{it}]^2$$

$$s.t. \quad l_i \geq 0$$

Solving the above nonlinear programming, the combination forecasting model can be obtained as follows:

$$x_{t+1} = \sum_{i=1}^n l_i x_{it}, t = 1, 2, \dots, N$$

**Forecasting model build**

Table 1 shows the sales of oil tank 2 of a petrol station in March 1, 2015 to 20. We forecast the sales volume of March 21, 2015 to 27 that the grey GM (1, 1) model and the BP neural network based on time series are used respectively programming by MATLAB2012b. The results are shown in Table 2

Table 1. Sales of oil tank 2 of a petrol station

date	sales of oil tank 2 (L)	date	sales of oil tank 2 (L)
2015/3/1	8989	2015/3/11	11697
2015/3/2	8743	2015/3/12	11937
2015/3/3	11129	2015/3/13	12188
2015/3/4	9798	2015/3/14	11372
2015/3/5	11152	2015/3/15	11427
2015/3/6	11048	2015/3/16	10667
2015/3/7	10729	2015/3/17	11620
2015/3/8	10453	2015/3/18	9469
2015/3/9	11592	2015/3/19	11729
2015/3/10	10496	2015/3/20	11919

Table 2. Results of GM (1, 1) model and BP neural network

date	results of GM (1, 1)	results of BP neural network
2015/3/21	11711	11374
2015/3/22	11784	12527
2015/3/23	11858	12121
2015/3/24	11932	11426
2015/3/25	12006	10511
2015/3/26	12081	11165
2015/3/27	12157	11431

Based on the method above, after solving the mathematical programming, the minimum value of the squares sum of error is 6.9497e-12, weight coefficients of GM (1, 1) model and the BP neural network are 0.2386 and 0.8376. Then the combination forecasting model is established as follows:

$$x_{t+1} = (0.2386x_{1,t+1} + 0.8376x_{2,t+1})\delta_{t+1}$$

$x_{t+1}$ ,  $x_{1,t+1}$  and  $x_{2,t+1}$  is the forecasting value of the combination forecasting model, GM (1, 1) model and the BP neural network.  $\delta_{t+1}$  is 0-1 discrete variables. When and only when the t+1 days petrol station is out of Service for special reasons (equipment maintenance, suspension, etc.)  $\delta_{t+1} = 0$ , otherwise,  $\delta_{t+1} = 1$ .

**DISCUSSION**

According to the historical sales data of oil tank 2 of the petrol station in March 1st to 26 (Table 1 and

table 3), we forecast sales from March 27, 2015 to 29, respectively by GM (1, 1) model ,the BP neural network and the combination forecasting model. The results are shown in Table 4

Table 3. Sales of oil tank 2 of a petrol station

date	sales of oil tank 2 (L)	date	sales of oil tank 2 (L)
2015/3/21	12321	2015/3/26	13152
2015/3/22	11262	2015/3/27	11269
2015/3/23	13515	2015/3/28	14100
2015/3/24	12388	2015/3/29	11759
2015/3/25	11771	-	-

Table 4. Results of GM (1, 1) model, BP neural network and combination forecasting model

date	real value	results of GM (1, 1) model	relative error	results of BP neural network	relative error	results of combination forecasting model	relative error
2015/3/27	11269.00	10869.00	3.55%	11034.00	2.09%	11835.42	5.03%
2015/3/28	14100.00	10854.00	23.02%	13842.00	1.83%	14183.82	0.59%
2015/3/29	11759.00	10839.00	7.82%	11242.00	4.40%	12002.48	2.07%
Avg.	12524.6	10839.4	11.46%	11551.4	2.77%	12261.73	2.56%

The average relative error of the GM (1, 1) model, is 11.46%, the BP neural network is 2.77%, and the combination forecasting model is 2.56%.

Thus, compared with each single prediction methods, the combination forecast method has higher precision. That means the combination forecast model is a relatively effective forecasting model. At the same time, consider that the prediction error can be as low as 0.59%, also as high as 5.03%, the volatility of sales is not fully reflected. The impact of external factors on the sales still exist.

Meanwhile, running the entire program requires nearly 3 minutes in forecasting sales of a single petrol station, which can be accepted. However, for a network containing 100 petrol stations, the model and the program will need to be improved.

**CONCLUSION**

In this paper, the grey GM (1, 1) model and BP neural network are combined to establish the combination forecast model. It has the advantages of requiring less information, a simple method need to build the GM (1, 1) model and strong nonlinear mapping ability, good fault tolerance, self-organization and self-adaptation of the BP neural network. The combination forecast model is applied in forecasting the sales of petrol and the error analysis is carried out with the results of each single demand forecasting model. The results show that the average error of the combined forecasting model is smaller than that of any single model, which indicates that the former forecasting model is effective.

However, directly or indirectly, to build the model based on historical data, the internal and external conditions of a complex system cannot be reflected well. Due to the insufficient reflection of the volatility of demand data, the reliability of the obtained prediction results needs to be improved. Therefore, factors, affecting the volatility of demand,

which is combined with the time series forecasting model, will be the focus of future research.

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