

Discussion on Local Resistance Coefficient of Sudden Expansion Pipe

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Abstract: It is introduced in the paper the theoretical derivation of local resistance coefficient of sudden expansion pipe. Combining the process of theoretical derivation, experiment device is designed for measuring local resistance coefficient. According to change of water head of pressure measuring pipe, optimal vena-contracta after sudden expansion pipe is correctly selected from Bernoulli equation. Through analysis of experiment data, it is pointed out that error of local water head loss calculated theoretically is more than 20%, while that calculated based on local resistance coefficient measured by experiment can almost be neglected. Therefore, in design of pipeline, local resistance coefficient of sudden expansion pipe measured through experiment is more close to the actual value.

Keywords Sudden expansion pipe; Local resistance; Coefficient

INTRODUCTION

On conveying pipeline of petroleum, some elbows, tees and reducer joints are installed for controlling and regulating flow of liquid in the pipe. While the liquid flowing through those accessories, uniform flow will be disturbed in the local area due to change of wall or flow rate, leading to change of value, direction or distribution of flow speed. The energy loss caused by the change is called local water head loss.

Due to inertia, when liquid flows from pipe with smaller diameter to pipe with bigger diameter, the flow will be expanded gradually rather than suddenly. Whirlpool will be formed between corner of pipe and main stream and rotates with the main stream. The rotation process will consume energy. Moreover, flow impact and collision etc. caused by sudden change of pipe section will bring loss of mechanical energy.

THEORETICAL DERIVATION OF LOCAL RESISTANCE COEFFICIENT OF SUDDEN EXPANSION PIPE

In Fig.1, take the space enclosed by internal wall of pipe, starting end of whirlpool and terminal end of whirlpool as control body, and apply Bernoulli equation between section 1 and sections, obtaining

$$z_1 + \frac{p_1}{\rho g} + a_1 \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + a_2 \frac{v_2^2}{2g} + h_{w1-2} \quad (1)$$

where, z is static head, $\frac{p}{\rho g}$ is pressure head, $\frac{v^2}{2g}$ is velocity head, a is kinetic energy correction factor, and h_{w1-2} is water head loss of liquid of unit gravity from section 1 and section 2.

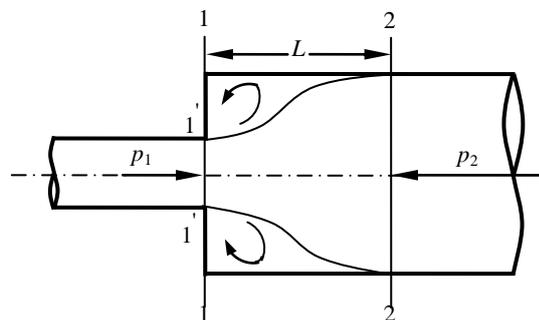


Fig.1 Scheme of Local Resistance of Sudden Expansion Joint and Control Body Selection

Since distance between the two sections is short, friction resistance can be neglected, i.e. head loss only includes local head loss h_j . Take $a_1=a_2=0$, obtaining

$$h_j = \frac{p_1 - p_2}{\rho g} + \frac{v_1^2 - v_2^2}{2g} \quad (2)$$

Apply momentum equation on control body. Since it is difficult to determine the pressure at section 1, it can only be assumed equal to pressure before the section. Therefore, total pressure on section 1 is $P_1=p_1A_2$ and total pressure on section 2 is $P_2=p_2A_2$. Neglecting side friction of control body, the joint force on liquid in the control body on flow direction is

$$\sum F = A_2(p_1 - p_2) \quad (3)$$

Therefore the momentum equation is

$$A_2(p_1 - p_2) = \rho Q(v_2 - v_1) \quad (4)$$

Two sides both divided by $\rho g A_2$, obtaining

$$\frac{p_1 - p_2}{\rho g} = \frac{v_2(v_2 - v_1)}{g} \quad (5)$$

Substitute it into formula (2), obtaining

$$h_j = \frac{v_2(v_2 - v_1)}{g} + \frac{v_1^2 - v_2^2}{2g} \quad (6)$$

Sort it and obtain

$$h_j = \frac{(v_1 - v_2)^2}{2g} \quad (7)$$

It can be obtained from continuity equation that

$$v_1 = \frac{v_2 A_2}{A_1} \quad \text{or} \quad v_2 = \frac{v_1 A_1}{A_2}$$

Formula (7) can be converted to

$$h_j = \left(1 - \frac{v_2}{v_1}\right)^2 \frac{v_1^2}{2g} = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{v_1^2}{2g} = \zeta_1 \frac{v_1^2}{2g} \quad (8)$$

or

$$h_j = \left(\frac{v_1}{v_2} - 1\right)^2 \frac{v_2^2}{2g} = \left(\frac{A_2}{A_1} - 1\right)^2 \frac{v_2^2}{2g} = \zeta_2 \frac{v_2^2}{2g} \quad (9)$$

Further obtaining

$$\zeta = \left(1 - \frac{A_1}{A_2}\right)^2 = \left(\frac{A_2}{A_1} - 1\right)^2 \quad (10)$$

ζ is the local resistance coefficient of sudden expansion pipe [Yang, et al., 2006]. It is shown from the formula that local resistance coefficient is only related to diameter ratio.

EXPERIMENT PLAN DESIGN

Experiment device is shown in Fig.2. Water feeding pipe on the left is connected to water supply pipeline, and feeding valve is used for control flow rate. Pipes used in the experiment is transparent organic glass pipe. Internal diameter of feeding pipe is 10mm, and diameter of expansion pipe is 16mm. Place the device levelly on the experiment bench. Water outlet is controlled by regulating valve.

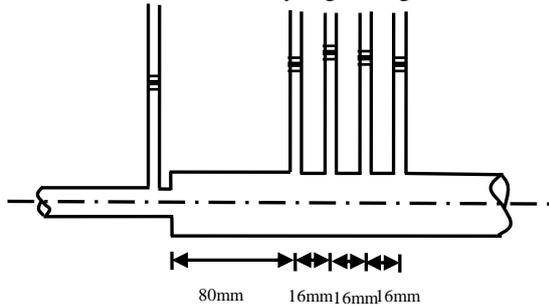


Fig.2 Experiment Device of Local Resistance Coefficient of Sudden Expansion Pipe

Positions of pressure measuring pipes: place first pressure measuring pipe at extreme position before sudden expansion; there is no accurate calculation formula for length of whirlpool formed by water flowing sudden expansion pipe, and it is commonly considered to be 5 to 8 times of diameter of sudden expansion pipe [Zhao et al., 1997]. Therefore place the other pressure measuring pipes at the positions of 80mm, 96mm, 102mm, 118mm after sudden expansion for analyzing change of energy loss.

Measurement of head of pressure measuring pipe: Fix transparent organic glass tube with height of 100mm and diameter of 6mm on lifting type pressure measuring plate. Minimum scale of the plate is mm.

Measurement of flow: volume will be adopted for measuring flow. Use counting pipe and counting cylinder for measuring volume and electronic stopwatch for timing.

ANALYSIS OF EXPERIMENT RESULT

Head line is commonly adopted to intuitively express energy change of main stream along pipeline. Due to viscosity of liquid, head loss is always existing along the pipe with monotonic decreasing; however, head of pressure measuring pipe reflects the potential energy of liquid, and the head line may be increasing, decreasing or staying same. For sudden expansion pipe, increment of water-carrying section will lead to decrement of average velocity and velocity head. Due to loss energy and decrement of velocity head of flow in the sudden expansion pipe, the total head line is decreasing; however, head line of pressure measuring pipe is increasing near the sudden expansion pipe [Zhao, et al., 2004]. Head line of pressure measuring pipe of frictional head loss is decreasing along the flow direction. Accordingly, optimal position of termination of whirlpool can be selected from the last four pressure measuring pipes, and equation can be established according to applying conditions of Bernoulli equation and section before sudden expansion.

It is shown from the experiment that with different velocity in the pipe, position of pressure measuring pipe at vena-contracta where streamline totally recovers is different. According to measured data, it is shown in table 1 the calculation of local head loss of sudden expansion pipe.

From Table 1, it is easy to find out that theoretically calculated head loss is rather higher. The reason is that friction head loss between the two sections is neglected for equation derivation. Experiment data proves that the error reaches 22.0%.

Therefore, local resistance coefficient of sudden expansion pipe shall be determined by following equation:

$$\zeta = \frac{h_j}{\frac{v_2^2}{2g}} \quad (11)$$

It is shown in Table 2 the calculation of local resistance coefficient of sudden expansion pipe.

It is known from the table that experiment measured value of local resistance coefficient of sudden expansion pipe is 3.126, significantly different from the theoretical value 2.434. Error of local head loss is only 0.2%.

CONSLUSION

In conventional calculation equation, local resistance coefficient is a constant. However, the experiment data shows that error of head loss calculated according to the conventional equation is significant, of which the main reason is incorrect assumptions and neglect of some conditions during

process of theoretical derivation. While designing pipeline, the design requirement can only be satisfied through measured section dimensions combining

with local resistance coefficient of sudden expansion determined by experiment data.

Table 1 Theoretical Calculation of Local Head Loss of Sudden Expansion

Flow rate (mL/s)	Velocity of sudden expansion pipe (m/s)	Pressure measuring height before sudden expansion (m)	Optimal pressure measuring height after sudden expansion (m)	Actually measured value of local head loss (m)	Theoretically calculate value of local head loss (m)	Error of local head loss (%)
173.858	0.865	0.715	0.809	0.118	0.093	-0.213
169.248	0.842	0.680	0.769	0.112	0.088	-0.213
164.104	0.817	0.667	0.753	0.103	0.083	-0.196
159.526	0.794	0.635	0.712	0.102	0.078	-0.229
154.772	0.770	0.599	0.677	0.090	0.074	-0.182
146.717	0.730	0.534	0.604	0.081	0.066	-0.183
129.531	0.645	0.435	0.489	0.064	0.052	-0.190
123.330	0.614	0.386	0.433	0.060	0.047	-0.217
107.727	0.536	0.302	0.335	0.048	0.036	-0.263
87.959	0.438	0.219	0.243	0.030	0.024	-0.214
68.918	0.343	0.136	0.148	0.021	0.015	-0.315

Table 2 Experiment Measuring Calculation of Local Head Loss of Sudden Expansion Pipe

Flow rate (mL/s)	Velocity of sudden expansion pipe (m/s)	Actually measured value of head loss (m)	Local resistance coefficient	Head loss calculated based on local resistance coefficient (m)	Error of local head loss (%)
173.858	0.865	0.118	3.092	0.119	0.011
169.248	0.842	0.112	3.094	0.113	0.010
164.104	0.817	0.103	3.026	0.106	0.033
159.526	0.794	0.102	3.158	0.101	-0.010
154.772	0.770	0.090	2.976	0.095	0.050
146.717	0.730	0.081	2.979	0.085	0.049
129.531	0.645	0.064	3.006	0.066	0.040
123.330	0.614	0.060	3.107	0.060	0.006
107.727	0.536	0.048	3.302	0.046	-0.053
87.959	0.438	0.030	3.097	0.031	0.009
68.918	0.343	0.021	3.552	0.019	-0.120

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