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A 2-phase Optimizing Strategy of Combining Weighting of Multiple Attribute Decision Making

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Abstract: In order to improve rationality and effectiveness of combining weighting of multiple attribute decision making, a 2-phase optimizing strategy was given. At the first phase of subjective weighting and objective weighting, the regression check of normal probability distribution was used to optimize the subjective weighting result, and the entropy theory and improved scatter degree method was combined to calculate objective weights based on the minimum deviation principle. At the second phase of combining weighting, the weights' minimum deviation of subjective and objective was used to realize the concordance of subjectivity and objectivity. In the end, an example was given to show the rationality and effectiveness of this 2-phase optimizing strategy.

Keywords multiple attribute, combining weighting, normal probability distribution, minimum deviation

INTRODUCTION

Multiple attribute decision making has been widely used in the fields of engineering design, economics, management, military operational and others [1-3]. In the process of multiple attribute decision making, the weights of attribute should be specified in advance[4]. At present, the method of attribute weighting could be divided into the subjective weighting method, the objective weighting method and the combining weighting method. The subjective weighting method means the weights of attribute was given by experts who may make arbitrary decisions for his lack of knowledge or experience. On the contrary, the weights of attribute was confirmed by analyzing decision matrix in the objective weighting method which means this process was based on the mathematical reasoning, and obviously it could not solve the problem of which the weighting result may deviate far from the decision maker's appetite. And the combining weighting method integrated the advantage of the subjective weighting method and the objective weighting method, so its weighting result was always more reasonable and effective to each of the pre-two method's weighting result.

It is obvious that the guarantee of effective and rational of combining weighting was based on the complementary program of subjective weighting and objective weighting, and plenty of research had been done with this object [5-14]. These research achievements could be summarized as transform the problem of combining weighting into the optimization of multi-objective according to the minimum deviation of subjective weights and objective weights or the accumulated deviation of the multiple attribute. And it had been proved well used in some specific questions. However, because of the complexity and variety of multiple attribute decision making problem, the combining weighting method should be studied deeply to improve its rationality and effectiveness.

In this paper, a 2-phase optimizing strategy was given. At the first phase of subjective weighting and objective weighting, the regression check of normal probability distribution was used in subjective weighting optimization which could help reduce the error made by the experts, and the entropy theory and improved scatter degree method was combined to be used to calculate objective weights based on the minimum deviation principle to realize the objective weighting optimization which could make use of the advantage of he entropy theory and the improved scatter degree method. At the second phase of combining weighting, the minimum deviation of subjective weight and objective weight was used to realize the concordance of subjective and objective to confirm this strategy could satisfy the requirement of subjectivity and objectivity.

OPTIMIZATION OF SUBJECTIVE WEIGHTING AND OBJECTIVE WEIGHTING

Optimization of subjective weighting

To the subjective weighting, the objectivity and credibility of weighting results was decided by the empiric level and intellectual differences of experts. And due to mathematical theory of statistics, the weighting results of one attribute made by experts could be hypothesized as a random variable. If the

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weighting result was reasonable and effective, the random variable should be normal distribution or approximating normal distribution. From the other point of view, if the weighting result was not normal distribution random variable or approximating normal distribution random variable, it means there may be some weighting results made by some experts was far from the true value.

According to aforementioned analysis, the hypothesis testing of normal probability distribution could be used to realize the optimization of subjective weighting. With the hypothesis testing of normal probability distribution, to a number of weighting results of one attribute made by experts, if they obeyed the normal distribution, the weighting results could be believed reasonable and effective, and the normal mean would be taken as the weighting results, the normal variance would be taken as the accuracy degree of subjective weighting. And if the weighting results did not obey the normal distribution, there may be some weighting results made by experts was wrong, and it should be rejected. Obviously, the weighting result with maximum deviation to the weighted average value should be rejected for it may be the most probable weighting result which had the utmost deviation to the true value. This procedure would be operated repetitive until the remaining variable obeyed normal distribution, and then, identically, the normal mean would be taken as the weighting results, the normal variance would be taken as the accuracy degree of subjective weighting.

To a multiple attribute decision making problem, the set of attribute could be hypothesized as $G=\{G_j \mid j=1,2, \ldots, n\}$, and the weighting matrix made by experts(the number of the experts was m)could be hypothesized as

$$W=\begin{bmatrix} \omega_{11} & \omega_{12} & \cdots & \omega_{1n} \\ \omega_{21} & \omega_{21} & \cdots & \omega_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \omega_{m1} & \omega_{m2} & \cdots & \omega_{mn} \end{bmatrix}$$

Taking the weighting results of the attribute q made by experts as the sample data, the hypothesis testing of normal probability distribution could be performed with the regression check of normal probability distribution [15].

If the variable ω_{iq} obey the normal distribution, the distribution function could be presented as:

$$F(\omega_{iq})=\int_{-\infty}^{\omega_{iq}}\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(\omega_{iq}-\mu)^2}{2\sigma^2}\right)d\omega_{iq} \quad (1)$$

And if

$$x_{iq}=\frac{\omega_{iq}-\mu}{\sigma},$$

$F(\omega_{iq})$ can be transformed to:

$$F(\omega_{iq})=\int_{-\infty}^{\frac{x_{iq}-\mu}{\sigma}}\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{x_{iq}^{\ 2}}{\sqrt{2}}\right)dx_{iq}=\Phi(x_{iq}) \quad (2)$$

So there was a linear relation between x_{iq} and ω_{iq}, and also the relationship between x_{iq} and $\Phi(x_{iq})$ was one-to-one correspondence. It means the regression check of normal probability distribution could be used to realize the hypothesis testing of normal probability distribution.

Therefore, the procedure of subjective weighting optimization could be operated as follows:

(1) Compare the numerical magnitude of weighting results, and make it to be an ascending sequence. For convenient, it was hypothesized that $\omega_{1q}<\omega_{2q}<\ldots<\omega_{iq}<\ldots<\omega_{mq}$ in this paper.

(2) Estimate the distribution function $F(\omega_{iq})$ and the standard normal distribution variable x_{iq}.

$F(\omega_{iq})$ could be calculated with middle rank

$$F(\omega_{iq})=(i-0.3)/(n+0.4) \quad (3)$$

For i was the ordering of the ascending sequence, and n was the number of sample data.

The standard normal distribution variable x_{iq} could be calculated with the method given in [16].

$$x_{iq}=r_{iq}-\frac{c_0+c_1r_{iq}+c_2r_{iq}^2}{1+d_1r_{iq}+d_2r_{iq}^2+d_3r_{iq}^3} \quad (4)$$

Where, $r_{iq}=\sqrt{\ln\frac{1}{(1-F(\omega_{iq}))^2}}$

c_0=2.515517, c_1=0.802853, c_2=0.010328,
d_1=1.432788, d_2=0.189269, d_3=0.001308.

(3) Linear regression analysis and correlation test of the variance.

According to the regression check of normal probability distribution, the linear relation between x_{iq} and ω_{iq} could be presented as:

$$x_{iq}=a\omega_{iq}+b \quad (5)$$

Where, $a=l_{\omega\omega}/l_{x\omega}$, $b=\overline{x}_{iq}-a\overline{\omega}_{iq}$,

$l_{x\omega}=\sum_{i=1}^{m}(x_{iq}-\overline{x_{iq}})(\omega_{iq}-\overline{\omega_{iq}})$,

$l_{\omega\omega}=\sum_{i=1}^{m}(\omega_{iq}-\overline{\omega_{iq}})^2$, $l_{xx}=\sum_{i=1}^{m}(x_{iq}-\overline{x_{iq}})^2$,

$\overline{x_{iq}}=\frac{1}{m}\sum_{i=1}^{m}x_{iq}$, $\overline{\omega_{iq}}=\frac{1}{m}\sum_{i=1}^{m}\omega_{iq}$.

And the linearly dependent coefficient

$$R_{x\omega}=l_{x\omega}/\sqrt{l_{\omega\omega}\cdot l_{xx}} \quad (6)$$

If the linearly dependent coefficient $R_{x\omega}$ was greater than or equal to the minimum dependent coefficient R in the confidence level of α, formula (5) holds. It means the variance ω_{iq} obey the normal distribution in the confidence level of α. The normal mean μ taken as the weighting result and the normal variance taken as the accuracy degree of subjective weighting σ could be calculated according to the linear regression analysis.

$$\mu=\frac{l_{x\omega}\overline{x_{iq}}-l_{\omega\omega}\overline{\omega_{iq}}}{l_{\omega\omega}},\quad \sigma=\frac{l_{x\omega}}{l_{\omega\omega}} \quad (7)$$

(4) Optimization of the subjective weighting results

If the linearly dependent coefficient $R_{x\omega}$ was less than the minimum dependent coefficient R in the confidence level of α, formula (5) was unsustainable. It means the variance ω_{iq} did not obey the normal distribution, and the subjective weighting results should be optimized.

For the weighting result that has the maximum deviation to the weighted average value may be the most probable weighting result had the utmost deviation to the true value, the method of rejecting the weighting result with maximum deviation to the weighted average value one by one until the remaining variable obey the normal distribution was used to realize the optimization in this paper. After the optimization, the normal mean μ was also taken as the weighting result and the normal variance σ was taken as the accuracy degree of the subjective weighting.

(5) Normalization of the optimization results

After the optimization of the subjective weighting of each attribute, the optimization results should be normalized. For the attribute set G_j, hypothesize the normal mean of the regression check of normal probability distribution of each attribute was $\mu_1, \mu_2, \ldots, \mu_n$, the optimized subjective weighting results ω_{Sj} (j=1, 2, …,n) could be presented in formula (8).

$$\omega_{Sj} = \frac{\mu_j}{\sum_{j=1}^{n} \mu_j} \tag{8}$$

Optimization of objective weighting

The regulating action of objective weighting in the combining weighting is that the variance of attribute index could be reflected in the evaluation result. Therefore, the entropy theory was used in the objective weighting for it take the variable quantity of the attribute index as the evaluated criterion, and the improved scatter degree method was used in the objective weighting for it take the maximum departure of the different suggestion as the evaluated criterion. In this paper, the method of combing the entropy theory and the improved scatter degree method on the principle of minimum deviation was given, which makes not only the variance of attribute index but also the maximum departure of the different suggestion principle could be reflected in the evaluation result. This method could be performed as follows:

Hypothesis the suggestion set which was being evaluated was $\{A_i\}$, (i=1, 2, …, m), and the index set which was being evaluated was $\{X_j\}$, (j=1, 2, …, n). x_{ij} was the primary data of the index j in the suggestion i. The weighting result with the method of the entropy theory was ω_{AOj}, and the weighting result with the improved scatter degree method was ω_{IOj}. According to the principle of minimum deviation, the optimization model of the objective weighting could be presented in formula (9).

$$\begin{cases} \min \sum_{j=1}^{n} (\omega_{Oj} - \omega_{AOj})^2 + (\omega_{Oj} - \omega_{IOj})^2 \\ s.t. \sum_{j=1}^{n} \omega_{Oj} = 1 \\ \omega_{Oj} > 0, j = 1, 2, \ldots, n \end{cases} \tag{9}$$

Solve formula (9):

$$\omega_{Oj} = \frac{\omega_{AOj} + \omega_{IOj}}{2}.$$

ω_{IOj} was the optimization result of the objective weighting.

OPTIMIZATION OF COMBINING WEIGHTING

Proportional relation of subjective weights and objective weights was calculated according to the objective function which was built based on the principle of minimum deviation with the combining weight or maximum departure of the different suggestion in the classical combining weighting method, which comes with the question of the objective information was overactive. In this paper, an objective function based on the principle of minimum deviation between the subjective weights and objective weights was built to realize the optimization of combining weighting which would make the subjective decision and objective information more concordance in the final decision making.

Based on this method, hypothesis ω_{Sj} and ω_{Oj} were optimized subjective weight and objective weight. The proportional factor of subjective weight (α) and objective weight (β) could be solved according to the optimization model (10). And the optimized combining weight could also be solved in formula (11).

$$\begin{cases} \min \sum_{j=1}^{n} (\alpha\omega_{Sj} - \beta\omega_{Oj})^2 \\ s.t. \alpha + \beta = 1 \\ \alpha \geq 0, \beta \geq 0, j = 1, 2, \ldots n \end{cases} \tag{10}$$

$$\omega_j = \alpha\omega_{Sj} + \beta\omega_{Oj} \tag{11}$$

Where,

$$\alpha = \frac{\sum_{j=1}^{n} \omega_{Oj}}{\sum_{j=1}^{n} (\omega_{Sj} + \omega_{Oj})}, \quad \beta = 1 - \alpha 。$$

ANALYSIS OF EXAMPLE

For indicating the rationality and effectiveness of this 2-phase optimization strategy, the example in paper [10] was used. In this example, the attribute G_1~G_4 were all effectiveness index, the suggestion set was A_1~A_8, and the decision matrix was R which was transform to Z by normalizing.

$$R=\begin{pmatrix} 8 & 24 & 44 & 0.3 \\ 7 & 20 & 43 & 0.4 \\ 4 & 18 & 37 & 0.2 \\ 4 & 19 & 36 & 0.2 \\ 5 & 20 & 40 & 0.3 \\ 4 & 19 & 36 & 0.3 \\ 3 & 18 & 37 & 0.2 \\ 5 & 20 & 39 & 0.4 \end{pmatrix} \quad Z=\begin{pmatrix} 0.5394 & 0.4279 & 0.3978 & 0.356 \\ 0.4719 & 0.3566 & 0.3887 & 0.4747 \\ 0.2697 & 0.3209 & 0.3345 & 0.2374 \\ 0.2697 & 0.3387 & 0.3254 & 0.2374 \\ 0.3371 & 0.3566 & 0.3616 & 0.356 \\ 0.2697 & 0.3387 & 0.3254 & 0.356 \\ 0.2023 & 0.3209 & 0.3345 & 0.2374 \\ 0.3371 & 0.3566 & 0.3526 & 0.4747 \end{pmatrix}$$

In the subjective weighting，the weights of each attribute made by three experts were: ω_{S1}=(0.3, 0.4, 0.15, 0.15), ω_{S2}=(0.4, 0.3, 0.15, 0.15), ω_{S3}=(0.25, 0.25, 0.25, 0.25).

According to the optimization strategy in this paper, the optimized subjective weights was ω_S=(0.3167, 0.3167, 0.1833, 0.1833).

According to the entropy theory in the objective weighting, the proportional matrix of different index was P. The entropy of each index was E=(1.4657, 1.4945, 1.4980, 1.4734). Therefore the weight of each attribute calculated based on the entropy method was ω_{AO} = (0.2411, 0.2560, 0.2578, 0.2451).

$$P=\begin{bmatrix} 0.2000 & 0.1652 & 0.1410 & 0.1304 \\ 0.1750 & 0.1246 & 0.1378 & 0.1739 \\ 0.1000 & 0.1121 & 0.1186 & 0.0870 \\ 0.1000 & 0.1183 & 0.1153 & 0.0870 \\ 0.1250 & 0.1246 & 0.1282 & 0.1304 \\ 0.1000 & 0.1183 & 0.1154 & 0.1304 \\ 0.0750 & 0.1121 & 0.1186 & 0.0870 \\ 0.1250 & 0.1246 & 0.1250 & 0.1739 \end{bmatrix}$$

And the weighting model made based on the improved scatter degree method was

$$\begin{cases} \max \omega^T Z^T Z \omega \\ s.t. \omega^T \omega = 1 \\ \omega > 0 \end{cases} \tag{12}$$

Where,

$$Z^T Z=\begin{bmatrix} 1.0001 & 0.9979 & 0.9722 & 0.9682 \\ 0.9979 & 1.0405 & 1.0165 & 0.9872 \\ 0.9722 & 1.0165 & 1.0000 & 0.9742 \\ 0.9682 & 0.9872 & 0.9742 & 1.0000 \end{bmatrix}$$

So the weighting results based on the improved scatter degree method was ω_{IO} = (0.2481, 0.2548, 0.2493 0.2475).

According to the objective optimization strategy, the optimized objective weights was ω_O=(0.2466, 0.2544, 0.2537, 0.2463).

Therefore, based on the 2-phase optimization strategy given in this paper, the final combining weights of each attribute was ω=(0.2807, 0.2861, 0.2185, 0.2148). And the evaluation value of the suggestion was A_1=0.4501, A_2=0.4214, A_3=0.2916，A_4=0.2947, A_5=0.3521, A_6=0.3202, A_7=0.2727, A_8=0.3757. It means the sequence of the suggestion was $A_1> A_2> A_8> A_5> A_6> A_4> A_3> A_7$.

This example could also been calculated based on the classical combining weighting method given in the paper of [17], [18] and [10].

With the method given in paper [17], the weighting result was ω=(0.4153, 0.114, 0.1052, 0.3655), and the evaluation value of the suggestion was A_1=0.4448, A_2=0.451, A_3=0.2705, A_4=0.2716, A_5=0.3488, A_6=0.315, A_7=0.2426, A_8=0.3912. So the sequence of the suggestion was $A_2 > A_1> A_8> A_5> A_6> A_4> A_3> A_7$. It means the evaluation result was almost consistent with the result calculated based on the 2-phase optimization strategy given in this paper expect for the suggestion of A_1 and A_2.

The weighting result was ω=(0.2847, 0.2587, 0.2167, 0.2937), and the evaluation value of the suggestion was A_1=0.4359, A_2=0.4247, A_3=0.2893, A_4=0.2919, A_5=0.352, A_6=0.3203, A_7=0.2701, A_8=0.3785 based on the method given in paper [18]. So the sequence of the suggestion was $A_1> A_2> A_8> A_5> A_6> A_4> A_3> A_7$, which was the same as the result calculated based on the 2-phase optimization strategy given in this paper.

Identically, the sequence of the suggestion was $A_1> A_2> A_8> A_5> A_6> A_4> A_3> A_7$ based on the method given in paper [10], which was the same as the result calculated based on the 2-phase optimization strategy given in this paper. And the weighting result was ω=(0.3114, 0.2617, 0.2046, 0.2223), the evaluation value of the suggestion was A_1=0.4405, A_2=0.4253, A_5=0.352，A_6=0.3183, A_8=0.3760.

Therefore, the 2-phase combining weighting strategy given in this paper was efficient in the multiple attribute decision making.

Moreover, from the example, we could see that the diversity between the optimum suggestion (A_1) and suboptimal suggestion (A_2) would be greater calculated with the strategy given in this paper than others in paper [10], [17] and [18] (The algebraic difference between A_1 and A_2 separate were 0.0062, 0.0112 and 0.152 based on the weighting method given in [17], [18] and [10], And with the strategy given in this paper, it was 0.0287). So the 2-phase optimizing strategy was more reasonable for it could distinguish the optimum suggestion (A_1) and suboptimal suggestion more easily (A_2).

CONCLUSION

A 2-phase optimization strategy of combining weighting was introduced. At the first phase of subjective weighting and objective weighting, the linear regression check of normal distribution was used to improve the rationality and effectiveness of subjective weighting. And the optimization of objective weighting was realized by building a multiple objective function based on the minimum variance of weight calculated by the entropy method and the improved scatter degree method in the objective weighting. At the second phase of weighting, the minimum variance of weight optimized in the subjective weighting and objective weighting was used to build the optimization function. For the 2-phase optimization strategy had improved the consistence of the subjective and objective of combining weighting, it would be more efficient and effective, and it had been proved in the example.

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