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A Super-Resolution Algorithm Based on Adaptive Total Variation Regularization

Ling Tang

School of Automation and Electronic Information, Sichuan University of Science & Engineering, Zigong 643000, China

Abstract: An algorithm with L_1 and L_2 mixed norm and bilateral total variation(BTV) regularization is proposed in this paper for image super-resolution. First, the mixed norm is used as the constraint of image fidelity; Secondly, considering the effect of the BTV method is not ideal for reconstruction in the edge and texture region, an adaptive regularization parameter algorithm is proposed. In the proposed algorithm, the local structure information of image is used to control the shape and the regularization parameter, which can be adjusted adaptively according to the local structure information of the image; Finally, a minimum gradient descent algorithm is used to update the algorithm. Experimental results show that the proposed algorithm can not only reduce the mean square error, improve the peak signal to noise ratio, but also can effectively smooth Gaussian noise and salt and pepper noise, maintain the image edges and texture details.

Keywords Super-resolution reconstruction; L_1 norm; L_2 norm; Adaptive regularization; Bilateral total variation

INTRODUCTION

The spatial resolution is a key index to evaluate the quality of the image, due to the limitations of hardware and cost, many imaging systems are not up to the requirements of high resolution(HR). The super-resolution(SR) image reconstruction technique becomes an important method to improve the resolution of the image [Park et al., 2003, Lucchese et al.,2000]. Thus the SR reconstruction is a typical ill posed problem, which is obtained from the low resolution(LR) image. This problem can be solved effectively by using the regularization method. Regularization technology means that reconstruct the stable and unique HR image by minimizing the objective function including the fidelity term and regularization term.

The regularization method is usually divided into Tikhonov regularization [AN et al. 2012], variation regularization, and total variation(TV) regularization method [Li et al., 2010]. At present, TV method is widely used which can effectively smooth the noise and eliminate the ambiguity. But the TV algorithm will generate the step effect in the smooth region with the strong noise, and not good to maintain the texture information of the image. Then Farsiu proposed a SR reconstruction algorithm based on bilateral total variation (BTV) [Farsiu et al., 2004], which can maintain good image edge details. Literature [Paul et al.,2009] proposed an iterative weighted TV reconstruction algorithm based on L_1 norm, and an image SR reconstruction algorithm based on L_1 and L_2 mixed norm [Li et al., 2015], which is used as the constraint of image fidelity and removes Gaussian noise and salt and pepper noise. However, most of these algorithms do not consider the local structure of the image, the effect of the reconstructed image in the edge and texture region is not ideal. So this paper proposed an adaptive BTV algorithm based on the mixed norm. First, the fixed kernel function is replaced by a controllable kernel function, which means the bandwidth and orientation of the kernel function changed with the local texture features of the image; Secondly, the adaptive regularization parameter is used to replace the fixed parameters, avoiding the subjectivity of the value.

RECONSTRUCTION ALGORITHM

Degradation Model

Consider N LR images $\{y_k\}_{k=1}^{N}$ each of size $M_1 \times M_2$, X is the desired HR image with size of $rM_1 \times rM_2$, r is sampling factor. The LR image sequences are generated from the degradation of the HR image. Thus the degradation model is expressed as follow:

$$y_k = \boldsymbol{D}\boldsymbol{B}_k\boldsymbol{M}_k X + n_k, \quad k = 1,2,3,\cdots,N \tag{1}$$

Where, $\boldsymbol{D}$ is a subsampling matrix, $\boldsymbol{B}_k$ is a blur matrix, $\boldsymbol{M}_k$ represents a warp matrix, and n_k represents the noise.

According to the degradation model, the image SR reconstruction can be converted to solve the following minimization problem:

$$\hat{X} = \arg\min[\sum_{k=1}^{N} \|y_k - \boldsymbol{D}\boldsymbol{B}_k\boldsymbol{M}_k X\|_p^p], \quad p = 1,2 \tag{2}$$

which is the fidelity term that describes the proximity of the LR image sequences and the HR image. The

Corresponding Author: Ling Tang, School of Automation and Electronic Information, Sichuan University of Science & Engineering, Zigong 643000, China

HR image X and the estimation $\hat{X}$ are closer means the degraded factors are more suitable for the practical conditions.

Regularization Reconstruction Algorithm

In order to obtain the stable solution and eliminate the noise, regularization term is usually required to be introduced in the cost function so as to constraint the solution space, that is:

$$\hat{X} = \arg\min[\sum_{k=1}^{N} \| y_k - \boldsymbol{D}\boldsymbol{B}_k\boldsymbol{M}_k X \|_p^p + \lambda J(X)] \qquad (3)$$

Where, $J(x)$ is the regularization term, λ is the regularization parameter, which is used to adjust the weight of the fidelity term and regularization term. When p=1 the fidelity term with L_1 norm, when p=2 the fidelity term with L_2 norm. The cost function respectively can be expressed as:

$$f_1 = \arg\min[\sum_{k=1}^{N} \| y_k - \boldsymbol{D}\boldsymbol{B}_k\boldsymbol{M}_k X \|_1^1 + \lambda J(X)] \qquad (4)$$

$$f_2 = \arg\min[\sum_{k=1}^{N} \| y_k - \boldsymbol{D}\boldsymbol{B}_k\boldsymbol{M}_k X \|_2^2 + \lambda J(X)] \qquad (5)$$

Usually L_2 norm is used to smooth the image with Gauss noise model. However, if the system noise is not Gauss, L_2 norm reconstruction algorithm is not robust. Because in the actual LR image sequences the noise is usually unknown, non-Gauss noise model is more suitable for practical SR reconstruction. So the L_1 norm reconstruction algorithm is more robust which is applicable to different noise models. But it is nonlinear and converges slowly. So we take L_1 and L_2 mixed norm algorithm to achieve SR reconstruction, that is:

$$\hat{X} = \arg\min[\mu\sum_{k=1}^{N} \| y_k - \boldsymbol{D}\boldsymbol{B}_k\boldsymbol{M}_k X \|_1^1 + (1-\mu)\sum_{k=1}^{N} \| y_k - \boldsymbol{D}\boldsymbol{B}_k\boldsymbol{M}_k X \|_2^2 + \lambda J(X)] \qquad (6)$$

Where, μ is the adjustment parameter of L_1 and L_2 norm.

The regularization term is usually a constraint on the nature of the image. BTV regularization term based on the bilateral filter can maintain the edge information while keeping the image smooth.

$$J_{BTV}(X) = \sum_{l=-q}^{q}\sum_{m=-q}^{q} \alpha^{|m|+|l|} \| X - S_x^l S_y^m X \| \qquad (7)$$

Where, q is the radius of the selection window, S_x^l and S_y^m represents l and m pixels shift along x and y directions respectively, α is the scale weighted coefficient with range of (0,1). According to equation (6) and (7) BTV regularization function based on mixed norm is obtained, as shown in formula (8):

$$\hat{X} = \arg\min[\mu\sum_{k=1}^{N} \| y_k - \boldsymbol{D}\boldsymbol{B}_k\boldsymbol{M}_k X \|_1^1 + (1-\mu)\sum_{k=1}^{N} \| y_k - \boldsymbol{D}\boldsymbol{B}_k\boldsymbol{M}_k X \|_2^2 + \lambda\sum_{l=-q}^{q}\sum_{m=-q}^{q} \alpha^{|m|+|l|} \| X - S_x^l S_y^m X \|] \qquad (8)$$

IMPROVED ALGORITHM BASED ON ADAPTIVE REGULARIZATION

Cost Function

Usually α in BTV regularization function is a constant, which is not reasonable in practice. The shape and parameter of the kernel function should be changed adaptively according to the image edge. On the basis of BTV method, the regularization function and parameters are improved in this paper, so as to be changed with the local characteristics of the image.

We use the controllable core given in the literature [Takeda et al., 2007] to replace the scale weighted coefficient, that is:

$$K_{H_i^s}(X_i - X) = \frac{\sqrt{\det(C_i)}}{2\pi h^2 \mu_i^2} \exp\{-\frac{(X_i - X)^T C_i (X_i - X)}{2h^2\mu_i^2}\} \qquad (9)$$

Where, C_i is the covariance matrix of the local gray values, h is smoothing parameter, μ_i is the local sample density parameter. From the literature, C_i determines the scale parameter γ_i , rotation parameter θ_i and tensile parameter σ_i of the kernel function. So the regularization function constructed is expressed as follow [JIANG et al., 2014]:

$$J_{BTV}(X) = \sum_{l=-q}^{q}\sum_{m=-q}^{q} K_{H_i^s}(V) \| X - S_x^l S_y^m X \| \qquad (10)$$

Where, V is the relative displacement which is l and m in x and y coordinate respectively. The kernel function is constructed according to the dominant direction of the local image data in the image, and the weight of the neighborhood pixels can be reasonably determined, so we can better preserve the edge information of the image. Insert equation(10) to(8),

$$\hat{X} = \arg\min[\mu\sum_{k=1}^{N} \| y_k - \boldsymbol{D}\boldsymbol{B}_k\boldsymbol{M}_k X \|_1^1 + (1-\mu)\sum_{k=1}^{N} \| y_k - \boldsymbol{D}\boldsymbol{B}_k\boldsymbol{M}_k X \|_2^2 + \lambda\sum_{l=-q}^{q}\sum_{m=-q}^{q} K_{H_i^s}(V) \| X - S_x^l S_y^m X \|] \qquad (11)$$

Adaptive Regularization Parameter

The regularization parameter is used to balance the regularization term and fidelity term, which directly impacts the reconstruction results. According to the literature [Moon et al., 1995, CHEN et al., 2011], the adaptive regularization parameter method is adopted:

$$\lambda(X_n) = \frac{\mu\sum_{k=1}^{N} \| y_k - \boldsymbol{D}\boldsymbol{B}_k\boldsymbol{M}_k X_n \|_1^1 + (1-\mu)\sum_{k=1}^{N} \| y_k - \boldsymbol{D}\boldsymbol{B}_k\boldsymbol{M}_k X_n \|_2^2}{\rho - \sum_{l=-q}^{q}\sum_{m=-q}^{q} \alpha^{|m|+|l|} \| X_n - S_x^l S_y^m X_n \|} \qquad (12)$$

Where, ρ is the controlling parameter, which satisfies $\rho \geq 2\sum_{l=-q}^{q}\sum_{m=-q}^{q} \alpha^{|m|+|l|} \| X_n - S_x^l S_y^m X_n \|$.

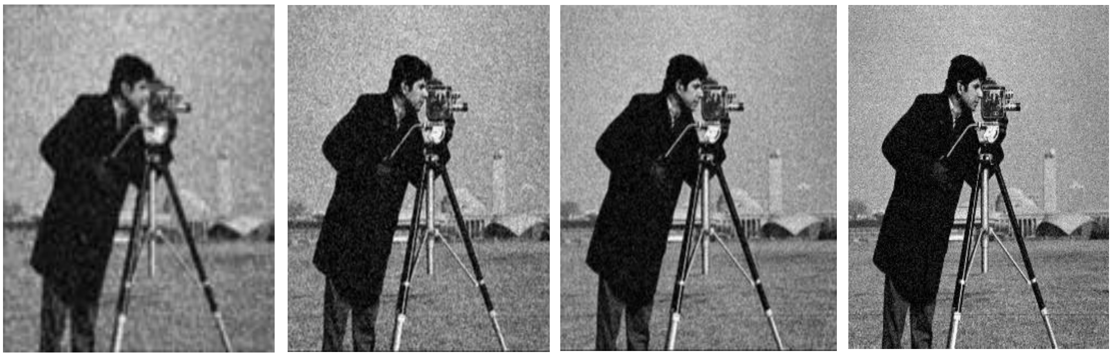
At last the iterative solution is carried out by using the minimum gradient descent method to obtain the HR image.

RESULTS AND DISCUSSION

In order to verify the effectiveness of the SR reconstruction algorithm with L_1 and L_2 mixed norm and adaptive BTV regularization, experiment on two groups of images. In the simulations, the downsampling factor is 2. Six LR images are generated after each group of the image is translated, blurred, sampled with noise added. The ambiguity function is Gauss kernel function which size is 3×3 and the variance is 1.

Experiment 1 Select the “Cameraman” image with a resolution of 256 x 256, SR reconstruction based on BTV algorithm with L_1 norm, with L_1+L_2 norm and the proposed adaptive BTV algorithm is respectively used to be compared on the reconstruction performance.

Fig.1 (a) is a LR image, Fig.1 (b) is the reconstruction effect using L_1 norm, Fig.1 (c) is the reconstruction effect using L_1+L_2 mixed norm, Fig.1 (d) is the result of the proposed algorithm. From the subjective vision, the proposed algorithm is softer and clearer with the other two algorithms, and the edge details are also very good.



（a）LR image （b）L_1 norm （c）L_1+ L_2 norm （d）The proposed algorithm

Figure 1. Comparison of different algorithms

Mean square error(MSE)、edge strength and peak signal to noise ratio (PSNR) as the objective index are evaluated in the experiment. The greater the MSE is, the more noise in the image, otherwise the better denoising effect. The greater the PSNR is, the better quality of the image reconstruction. The objective index evaluations are shown in Table 1.

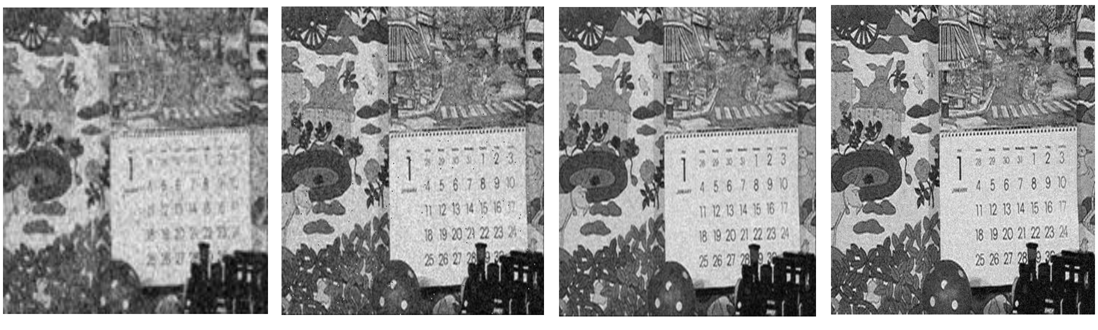
Table 1. Comparison of objective index evaluations

Algorithm	MSE	PSNR（dB）	Edge strength
L_1 norm	30.56	32.42	60.79
L_1+L_2 norm	18.24	33.19	60.65
Proposed algorithm	17.18	35.63	61.02

From Table 1, the proposed algorithm can improve the PSNR by 2.4dB than the mixed norm algorithm, and 2.4dB than the L_1 norm method. The MSE is reduced from 30.56 to 17.18, the edge strength is also increased. So the quality of the image is improved.

Experiment 2 Adding Gauss noise and salt and pepper noise to verify the effect of the SR reconstruction algorithm with mixed norm and adaptive BTV. Select the “calendar” image of size 352×288. Simulation results are shown in Fig.2.

Fig.2 (a) is a LR image adding Gauss noise and salt and pepper noise, Fig.2 (b) is the reconstruction effect using L_1 norm, Fig.2 (c) is the reconstruction effect using L_1+L_2 mixed norm, Fig.2 (d) is the result of the proposed algorithm. It can be seen that the quality of the reconstructed image is significantly enhanced using the proposed algorithm, which can effectively remove the noise and improve the reconstruction effect, the edge details also remain good.



（a）LR image （b）L_1 norm （c）L_1+ L_2 norm （d）The proposed algorithm

Figure 2. Comparison of different algorithms adding salt and pepper noise

CONCLUSION

Taking into account the Gauss noise and salt and pepper noise in the image, the advantages and disadvantages of L_1 and L_2 norm, an adaptive BTV super-resolution reconstruction algorithm based on the mixed norm is proposed in this paper. Using the controllable core as the weighted coefficient, adaptively determine the weight of the neighborhood pixels, and thus construct the regularization parameter which is adaptively adjusted with the local structure information of the image to make the value rational. Experimental results demonstrate the effectiveness and feasibility of the proposed algorithm, which can provide a good technical basis for vehicle identification, face recognition and video surveillance.

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