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The Model of Bathtub Water Temperature Control

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Abstract: In order to keep bath water closely to the start temperature, and the person could feel comfortable and relaxed all the time with not wasting so much water, we establish the model to study change of water temperature on adequate consideration of the shape/volume/ motion of the person in the bathtub.

First of all, without considering the factor of people and water, the convection and the thermal conductivity are the main factors. We establish the model of the water temperature control. Secondly, two models are established on the condition of water and people. In model one, people in the bathtub is stationary. The water is added to bathtub when the temperature dropped to the lowest, while no water entering until the water temperature rises to a given maximum temperature. Model two make the consideration of the movement of the people, other factors such as bubble agent. Finally, we summary for the model established and provide suggestions.

Keywords water temperature control; mathematical model; heat convection; thermal conductivity

INTRODUCTION

The field studying water temperature control are ecosystems, holothurian aquaculture, hot spring pool, cave drip etc. Mortsch LD and Bergstedt R study that Changes in water temperature have an important impact on ecosystems [Mortsch et al., 1996. Bergstedt et al., 2003]. A large number of water temperature models exist. They are often categorized in two main groups: deterministic and statistical approaches[Benyahya et al., 2003].Considering all kinds of factors, such as the cubage of the pond, the number and the distributing of the water inlets and outlets, the temperature of the water and the environment,some studyer established a mathematical model about the relationship of the temperature of water and the time to inject underground water into the pond according to the heat balance equation in the premise of making necessary simplification and assumption [Chen et al., 2003]. Adopting math idea is from simple to complex, from special to generic. In turn, Li jinghai have done some research on immobile hot spring pool and floating hot spring pool. According to the law of conservation of energy, he set up three differentia equation models regarding control the temperature diversification of the hot spring [Li et al., 2006]. To identify the processes that control the cave drip water temperature, Gabriel C measured the temperatures at multiple locations along a speleothem flow path and drip sources (stalactites)concurrently with the drip rates in Cathedral Cave, Wellington, Australia [Gabriel et al., 2015]. Cuthbert reported that cave drip water is only activated after long duration and high volume rainfall, and that evaporation from the epikarst is an important control on drip water isotopic composition [Cuthbert et al., 2015].

However little work has been done on what controls the temperature of bath water and yet this is of fundamental importance as it controls water temperature when we pursuit comfortable bath. As people's living condition, leisure life, pattern of consumption, bath values etc. changed. Now great changes have occurred in the way and request of bath. People pay more attention on the pursuit of more comfortable bath environment and atmosphere. It is necessary to study bath water temperature control in order to meet the needs of people and enhance people's convenience and comfort.

For the research in this essay, we are committed to building a more perfect bathtub water temperature model. In this model, we hope the temperature of the whole process of bath can keep closely to the start temperature, and the person could feel comfortable and relaxed all the time with not wasting so much water. Under the conditions both economical and environmental, to deal with people's all kinds of request for bath.

ASSUMPTIONS AND JUSTIFICATION

Assumptions

1)Water referenced herein are slow moving, regardless of their kinetic energy into heat.

2)Dose not consider the potential converse into heat of bathtub water due to the height difference arising from the import and export.

3)Does not consider the loss of heat of water at different temperatures in the mixing process

4)Does not consider heat loss in the process of water entering into bathtub as the distance of faucet and water surface is so small.

5)Does not consider evaporation of water from the bathtub.

6)Just consider radiating in water surface, ignore bathtub's.

7)Drain and faucet are located in two narrow walls, and far enough apart.

8)The body feels comfortable water temperature is 33 ℃ -37 ℃.

9)Initial air temperature around bathtub is 25 ℃.

Symbol Description

m : The initial quality of the water when reaching to critical line

c : The specific heat capacity of water

f_1 : Velocity of flow of water from faucet

ρ : The density of water

H_1 : The radiation coefficient of the outer wall of the bathtub

S_1 : Surface area of the bathtub

λ : Thermal Conductivity of the wall of bathtub

ε : Boltzmann's constant

T : The temperature of the water in the bathtub at time of t

H_2 : The heat transfer coefficient of water

σ_2 : Water emissivity

T_{max} : The temperature of hot water flowing into the bathtub

T_1 : The temperature of the external environment

T_2 : The temperature of the surface of the bathtub wall

D : The thickness of the wall of bathtub cement board

W : The value of the loss of heat energy guide at per unit of time due to water heat transfer

V_0 : The initial amount of water in the bathtub (Two-thirds of the critical line)

μ : The coefficient of people in the bathtub influencing water temperature changes at per unit time

V : The volume of people

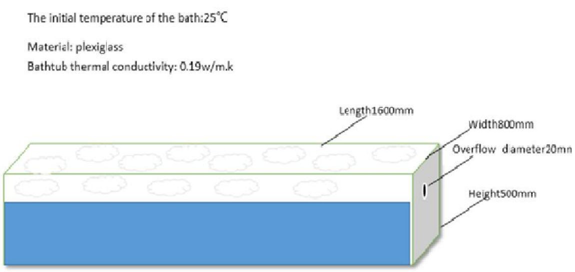

Figure 1 Schematic of bathtub

In this paper, the value of needed parameters is described as follows:

$T_0 = 36$℃ , $T_1 = 25$ ℃, $S_1 = 2.4m^2$,

$\lambda = 0.19w/m \bullet k$, $D = 0.02m$,

$H_2 = 0.626w/m \bullet k$, $S_2 = 1.28m^2$,

$C = 4.2\times 10^3 J/kg \bullet °c$, $m = 426.66kg$,

$\rho = 10kg/m^3$, $k = 0.02$,

$f_1 = f_2 = 0.04m^3/s$, $\mu = 0.9$

MODEL

Static type

There is no hot water enter the bathtub, no water discharge the bathtub in the bathroom, Apparently, the thermal energy of water in the bathtub exchange it's energy with the outside world through heat conduction, convection and radiation.

Assumptions:

1) Water at different temperatures in the mixing process does not consider the loss of heat.

2) Does not consider heat loss in the process of water entering into bathtub as the distance of faucet and water surface is so small.

3) Does not consider evaporation of water from the bathtub.

4) Just consider radiating in water surface, ignore that of bathtub.

5) Drain and faucet are located in two narrow walls, and far enough apart.

6) Initial air temperature around bathtub is 25 ℃.

7) No people in the bathtub

According to heat energy balance: the heat energy change of the water in the bathtub is equivalent to the heat loss of the water in the bathtub. It's easy to get the model:

$$cm\frac{dT}{dt} = -H_1S_1(T_2-T_1) - \frac{S_1\lambda(T-T_1)}{D} - \varepsilon\delta_1S_1(T_2^4-T_2^3) - H_2S_2(T-T_1) - \varepsilon\delta_2S_2(T_2^4-T_2^3) - W \tag{1}$$

Annotation: The left side of the equation is the instantaneous change of the heat energy of the bathtub water. For the right side of the equation ,the first item is the heat loss due to convection between the outer wall of the bathtub and the outside world; the second item is the heat loss due to heat conduction between the outer wall of the bathtub and the outside world; the third item is the heat loss due to radiation between the outer wall of the bathtub and the outside world; the fourth item is the heat loss due to convection between the upper-surface of bathtub water and the outside world; the fifth item is the heat loss due to heat radiation through upper-surface of the bathtub and the outside world; the sixth item is the heat loss due to heat conduction through the upper-surface of bathtub water and the outside world.

For the bathtub wall, the heat loss caused by convection and radiation is much less than the heat loss caused by the heat conduction. So both of them can be ignored. In the same way, for the upper-surface of the bathtub water, the heat loss caused by heat conduction and radiation is much less than the heat loss caused by convection. Also both of them can be ignored. So after the model can be simplified to get:

Model 1

$$\begin{cases} cm\dfrac{dT}{dt} = -\dfrac{S_1\lambda(T-T_1)}{D} - H_2S_2(T-T_1) \\ T(t=0) = T_0 \end{cases} \tag{2}$$

Using MATLAB software to solve the model one and obtaining the result as follows:

$$T = T_1 + e^{\frac{(s_1\lambda - h_2 s_2 D)t}{cdm}}(T_0 - T_1) \tag{3}$$

When the value of parameters are Substituted, we can figure out the graph as follows:

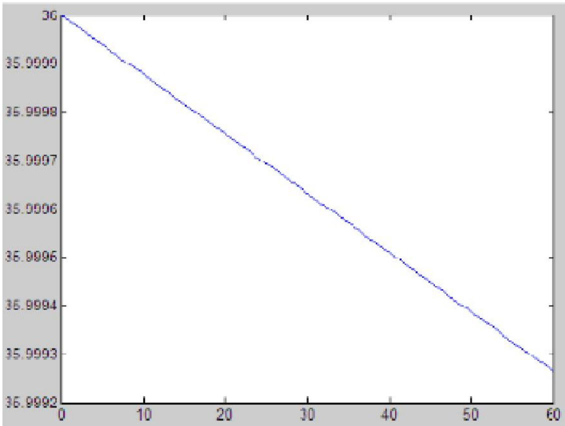

Figure 2 model one

When t in the open interval ranging from 0 to 30, the graph about the vary of temperature with time is described as above. In this case, not considering the element of person, the decreasing of temperature due to heat loss resulted from convection between the wall of bathtub and outdoor, the upper surface of bathtub and outdoor. We can conclude from above cartoon is that owing to the loss of energy causes the temperature continuing to decrease.

dynamic type

The thermal energy of water in the bathtub exchanges its energy with the outside world through heat conduction, convection and radiation. The heat energy released from the hot water into the bathtub is the difference between the amount of heat carried by the hot water entering the bathtub at a certain time and the amount of heat taken away by the water in the bathtub at the same time. The heat absorbed by the water into the bathtub is the difference between the heat taken away by the water in the bathtub at a certain period of time and the heat energy that enters the bathtub at the same time. In this condition, the water in the pool is constant.

Model 2

Assumption:

(1)The person is moving in the bathtub

(2)Water referenced herein is slowly moving, regardless of their kinetic energy into heat.

(3)Dose not consider the potential converse into heat of bathtub water due to the height difference arising from the import and export.

(4)Does not consider the loss of heat of water at different temperatures in the mixing process

(5)the person are not moving in the bathtub

$$\begin{cases} c(m-m_p)\dfrac{dT}{dt} = f_1\rho_c(T_0-T) - \dfrac{S_1\lambda(T-T_1)}{D} - H_2S_2(T-T_1) - kV \\ T(t=0) = T_0 \\ m_p = \rho V \end{cases} \tag{8}$$

Using MATLAB software to solve the model and obtaining the result as follows:

$$T = \left\{ \frac{1}{N} e^{\left[\frac{Nt}{cD(m-m_0)}\right]}(M-kVD) + T_0 - \frac{M-kVD}{N} \right\} e^{\frac{-Nt}{cD(m-m_0)}} \tag{9}$$

$$M = s_1\lambda T_1 + f_1\rho cDT_0 + h_2 s_2 DT_1 \tag{10}$$

$$N = cD\rho f_1 + s_1\lambda + Ds_2h_2 \tag{11}$$

When the value of parameters is substituted, we can obtain a graph as follows:

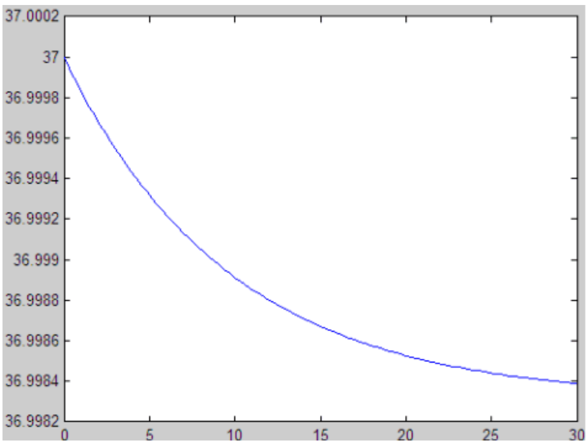

Figure 3 model two

In this model, the movement of the person entering the bathtub have effects on the speed of heat exchange. From the cartoon, we can see that the decreasing of water temperature of this model is more fast than the above one .But in the final, the vary of temperature gradually tends to balance with the change of time.

Model 3

If the person used a bubble bath additive while initially filling the bathtub to assist in cleansing.

Assumption:

(1)Using μ to describe the influence on coefficient of water when temperature changes at per unit time

(2)The person use a bubble bath salt while initially filling the bathtub to assist in cleansing.

(3)The person is moving in the bathtub

$$\begin{cases} c\left(m-m_p\right)\dfrac{dT}{dt}=f_1\rho_c(T_{\max}-T)-\dfrac{S_1\lambda(T-T_1)}{D}-uH_2S_2(T-T_1)-kV \\ T\left(t=0\right)=T_0 \\ m_p=\rho V \end{cases} \tag{12}$$

Using MATLAB software to solve the above model and obtaining the result as follows:

$$T=\left[\frac{A}{B}\bullet e^{\frac{Bt}{cD(m-m_0)}}+T_0-\frac{A}{B}\right]\bullet e^{\frac{-Bt}{cD(m-m_0)}} \tag{13}$$

$$A=cD\rho f_1T_0+s_1\lambda T_1+\mu h_2s_2DT_1-kvD \tag{14}$$

$$B=cD\rho f_1+s_1\lambda+\mu h_2s_2D \tag{15}$$

When the value of parameters are substituted, we can obtained a graph as follows:

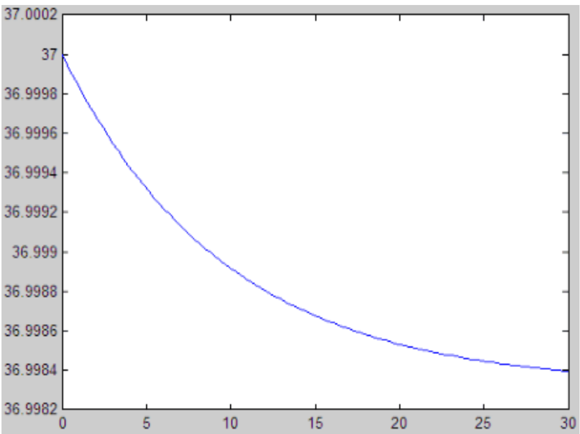

Figure 4 model three

When t in the close interval ranging from 0 to 30, Temperature changes over time, just as shown in above diagram. In this case, considering the movement of the person, the movement of water that entering the bathtub is equal to the amount of water that discharge the bathtub. Temperature drop is mainly caused by the heat loss which due to heat conduction caused between the wall of bathtub and environment, and the heat loss which due to heat conduction caused between the upper-surface and environment. At first, because the difference of temperature is large, the temperature dropped relatively fast, but with the decrease of temperature difference, the temperature fall more slowly. However, there is no balance, the temperature is also slowly decreased with time passing by.

Because of the presence of foam, resulting in the most of space of bathtub is occupied by the foam, leading to the time of heat loss longer than pure water.

Due to the bubble bath salt is mixed of sodium bicarbonate, sodium hydrogen carbonate and citric acid or tartaric acid, which leads to doped with trace amounts of sodium ions in the water, so leading to a pool of water in specific heat capacity and thermal conductivity changes. Finally, the cooling time and transit time are effected.

ANALYSIS OF MODEL AND SUGGESTION

This essay is based on the view of static and dynamic point, considering the influence of various factors on heat exchange. Firstly, a simple static base model is established, and then, the other factors are gradually considered to establish a general model. In the process of bath, except for the difficult control of hot loss, there is a greater impact on the temperature of the bath water and a number of other factors.

For the factors of time and space, we give the following suggestions:

(1) The upper-boundary temperature of water is 37℃.

(2) The controlling of hot water adding : The initial room temperature is 25℃, there is a large difference with the 37℃ of the hot water, the first time of water temperature drop from 37℃ to 34℃ is short, through the model function, we know that the time of first adding the heat water is about 3 minutes; in the following time, the indoor temperature will continue to absorb the heat water until the water of the bath's temperature reaches equilibrium with the indoor environment, this process also need to add water for two times, the time span between the second time of adding water and the first time of adding water is 4 minutes, After adding water three times, the indoor temperature and the bath temperature reaches the equilibrium, then we can add water after every 8 minutes. Through the analysis of the model, we suggest that we'd better control the bathing time in less than 45 minutes.

(3) The best selection of bathtub material is acrylic, the best shape of bathtub is square. From the perspective of comfortable and convenience, in the case of full water, the capacity of bathtub is 230~320L, the length is 1500mm, the width is 80mm, the deep is 50mm.

CONCLUSION

Through the setting of the above three models and the corresponding graph, we can find that in the static case, which the temperature decreased with time without taking into account the human factors. In dynamic case: First, when the volume of a person is certain, the temperature changes more quickly than before. Second, the greater the movement of people, the faster the temperature falls and then we add the bubble, the temperature drop become slowly. This is consistent with the fact, so we can conclude that the model is feasible. the water temperature balance is difficult to control, as well as describing how to control the temperature balance of the bath water. We hope it will be helpful to the bath users.

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