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Stochastic Operational Optimization for Metallurgical Blending Process Under Uncertainty

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Abstract: This paper presents a stochastic operational optimization method for metallurgical blending processes by focusing on the uncertainty of composition of raw materials, where the stochastic optimization model is firstly formulated by explicitly incorporating uncertain parameters into the constraints as random variables. Then an efficient sampling technique is utilized to construct the expectation counterpart of the stochastic model, in which Monte Carlo sampling is commonly employed. Finally, an HSS stochastic genetic algorithm is proposed to solving the stochastic optimization problem, where HSS technique is used the initial population generation and population updating so as to improve the population diversity and uniformity of random operations. The results show that the proposed method can guarantee the quality of blending products as well as greatly reduce the consuming cost of raw materials under uncertainty.

Keywords Blending operation, Stochastic optimization, HSS, Genetic algorithm

INTRODUCTION

The blending operation is an important process in many metallurgical industries such as alumina, copper and lead-zinc. The optimization control of blending operation not only can improve the quality of mixed products and reduce the cost of raw materials, but also has an important effect on the stabilization of subsequent processes and the energy consumption of whole production. Therefore, it is of great significance to effectively realize the optimization control of blending operation.

The operational optimization for blending process is of increasing importance and has attracted attentions of many researchers. Conventionally, the linear or nonlinear optimization is the favorite method to optimize the blending operation [1-2]. However, these methods only can effectively optimize the small-scale blending process with the accurate model. For the metallurgical blending process, many intelligent technologies such as neural networks [3-4] and expert system [5-6] have been used realize the operational optimization for the blending processes. Unfortunately, intelligent optimization has a high requirement of prior operation knowledge that highly depends on expert experience which may not be available [7]. Furthermore, these optimization approaches mentioned above may fail to guarantee a feasible solution under uncertainty. A main reason of this failure is the implicit assumption that data and parameters remain changed or are equal to their nominal values.

In fact, most of the real-world optimization problems are subject to various types of uncertainties. There are a large number of uncertainty sources in the blending process of metallurgical process industries due to instability of mine sources, data incompleteness and large time delay of measurements. These uncertainties result in the problem data that make an optimal operating obtained by the conventional optimization technologies without the consideration of uncertainty to be unfeasible at some level of probability. To deal with the uncertainties, a number of approaches have been proposed in engineering optimization applications including solution sensitivity/stability analysis, robust optimization and stochastic optimization. Sensitivity analysis was originally employed to measure the impact of changes in the problem data on the optimal solution. Robust optimization [7-8] is used to optimize the expected value of a chosen performance index for a given of risk and includes formulations based on expectation, weighted mean-variance, worst-case and so forth. Alternatively, stochastic optimization [9-10] attempts to directly solve a problem given uncertainty in the data. Since stochastic optimization deals directly with uncertainty in an explicit manner, it is a good candidate for the optimization of blending operation of metallurgical process industries with parametric uncertainty.

This paper focuses on the uncertainty of quality parameters of raw materials and present a stochastic optimization method based on an efficient sampling technique and genetic algorithm for the metallurgical blending process. The work firstly develops a stochastic optimization model where uncertain quality parameters are explicitly incorporated into the quality constraints. Then, an efficient sampling technique, i.e. Hammersley sequence sampling (HSS) [11], instead of Monte Carlo sampling that is commonly employed, is utilized to obtain the expectation counterpart of stochastic model. Finally,

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a Hammersley stochastic genetic algorithm is adopted to solve the the expectation optimization model, where HSS technique is used in the initial population generation and population updating so as to improve the population diversity and uniformity of random operations. The proposed method is validated by using production data collected from the actual alumina blending process. Experimental results obtained show that the proposed method can effectively reduce the cost of raw materials as well as guarantee quality.

The rest of this paper is organized as follows. Section 2 develops the stochastic optimization model of blending operation for metallurgical process industries. Section 3 presents the expectation counterpart of stochastic model by utilizing HSS technique. Hammersley stochastic genetic algorithm for blending problem is introduced in Section 4. Section 5 describes the experimental results obtained using real production process data. Some conclusions are made in Section 6.

STOCHASTIC OPTIMIZATION MODEL FOR THE BLENDING OPERATION

The expectation counterpart of stochastic model

In the blending operation of process industry, the quality indices of mixed products are usually required to control in a given ideal interval by regulating the raw material proportioning. These quality indices are not only related to the raw material proportioning, but also affected by the composition of raw materials. However, due to the instability of mine source, the time delay of composition measurement and environmental disturbance, it is difficult to obtain an accurate value for the composition of raw materials in real time. To deal with the uncertainties of composition, these quality parameters of raw materials are taken as random variables to enter the quality constrains of the optimization model. In addition to the quality requirements, the most immediate requirement is to reduce the cost of raw materials. Therefore, the stochastic optimization model for blending operation is developed by minimizing the cost of raw materials and the deviations between the expected interval goals and the actual values of the quality indices.

Suppose there are m kinds of raw materials and the number of quality indices is n; x_i is the feed rate of the ith kind raw material, c_i is the price of the ith raw material. Thus, $x_1, x_2, \cdots x_m$ are the decision variable.

Let

$$\boldsymbol{c} = (c_1, c_2, \cdots c_m)^{\boldsymbol{T}} \tag{1}$$

$$\boldsymbol{x} = (x_1, x_2, \cdots x_m)^{\boldsymbol{T}} \tag{2}$$

where $\boldsymbol{x}$ represents a scheme of raw material proportioning and where $\boldsymbol{c}$ is the cost vector of raw material.

Thus, the stochastic optimization model for blending operation optimization can be written as:

$$\min \quad F(\boldsymbol{x}) = \boldsymbol{c}^T \boldsymbol{x} + \sum_{k=i}^{n} \alpha_k (d_k^+ + d_k^-)$$

$$s.t. \begin{cases} d_k^+ = [\, f_k(\boldsymbol{x}, \tilde{\boldsymbol{\varepsilon}}) - \bar{b}_k \,] \vee 0, \\ d_k^- = [\underline{b}_k - f_k(\boldsymbol{x}, \tilde{\boldsymbol{\varepsilon}})] \vee 0, \\ d_k^+, d_k^- > 0, \\ \tilde{\boldsymbol{\varepsilon}} \in [\underline{\boldsymbol{\varepsilon}}, \bar{\boldsymbol{\varepsilon}}] \\ \sum_{i=1}^{m} x_i = M \\ x_i \geq 0 \\ i = 1, 2, 3, \cdots, \mathrm{m}; k = 1, 2, 3, \cdots, n. \end{cases} \tag{3}$$

where $\alpha_k \in [0,1]$ is the weighting factor of the kth quality index; $\underline{b}^k$ and $\bar{b}_k$ denote the lower limitation and the upper limitation of the kth quality index, respectively; d_k^+ and d_k^- stand for the positive deviation and the negative deviation between the expected interval goal and the actual value of the nth index, respectively; $\tilde{\boldsymbol{\varepsilon}}$ is the composition matrix of raw materials and taken as random variables, $\underline{\boldsymbol{\varepsilon}}$ and $\bar{\boldsymbol{\varepsilon}}$ represent its lower and upper boundary; M is the allowable maximum production; $f_k(\boldsymbol{x}, \bar{\boldsymbol{\varepsilon}})$ is a quality function which describes the relationship between the kth index and the composition and proportioning of raw materials.

Here,

$$d_k^+ = [\, f_k(\boldsymbol{x}, \tilde{\boldsymbol{\varepsilon}}) - \bar{b}_k \,] \vee 0 = \max[0,\, f_k(\boldsymbol{x}, \tilde{\boldsymbol{\varepsilon}}) - \bar{b}_k] \tag{4}$$

$$d_k^- = [\underline{b}_k - f_k(\boldsymbol{x}, \tilde{\boldsymbol{\varepsilon}})] \vee 0 = \max[\underline{b}_k - f_k(\boldsymbol{x}, \tilde{\boldsymbol{\varepsilon}}), 0] \tag{5}$$

HSS

For continuous parameter distribution, the sampling-based decomposition scheme is the most commonly used method to obtain the approximately equivalent expectation model. Here, instead of the Monte Carlo sampling (MCS) technology [12] usually resulting in large error bounds and variance, an efficient HSS technology is adopted to obtain the expectation counterpart of stochastic model for blending operation.

HSS is an efficient sampling technique which has an optimal design scheme for placing the n points on k-dimensional hypercube. The scheme ensures that the sample set is more representative of the population, showing more uniformity properties in multi-dimensions than the other sampling techniques [11].

Let R be a prime number. Then any integer n, $n \geq 0$,can be written in base- R representation as

$$n = n_m R^m + n_{m-1} R^{m-1} + \cdots + n_1 R + n_0 \quad (5)$$

where $n_i \in \{0,1,\cdots,R-1\}$, $i = 0,1,\cdots,m.$

Define the base-R radical inverse function, $\phi_R(k)$, as

$$\phi_R(n) = \frac{n_0}{R^1} + \frac{n_1}{R^2} + \cdots + \frac{n_m}{R^{m+1}} \quad (6)$$

Notice that for every integer $n \geq 0$, $\phi_R(n) \in [0,1]$.

The nth element of Hammersley sequence is obtained via the radical inverse function evaluated at n. Specially, if R_1, R_2, $\cdots$, R_{k-1} are $k-1$ different prime numbers, in a k-dimensional hypercube, Hammersley points can be denoted as

$$\boldsymbol{z_k}(n) = (\frac{n}{N}, \phi_{R_1}(n), \phi_{R_2}(n), \cdots, \phi_{R_{k-1}}(n)), \quad n = 1,2,\cdots,N. \quad (7)$$

Thus, a k-dimensional Hammersley sequence of length N is

$$\{\boldsymbol{z}_k(1), \boldsymbol{z}_k(2), \cdots, \boldsymbol{z}_k(N)\} \quad (8)$$

Approximate expectation model

According to the characteristics of alumina blending process, the random vector $\tilde{\boldsymbol{\varepsilon}}$ is assumed to follow the uniform distribution on the closed interval $[\underline{\boldsymbol{\varepsilon}}, \overline{\boldsymbol{\varepsilon}}]$.Supporting that the composition matrix of raw materials $\tilde{\boldsymbol{\varepsilon}}$ has K non-zero elements in a stochastic optimization model (3), the corresponding expectation model was achieved by adopting HSS technology as follows:

Let $j = 1$ and take $K-1$ prime numbers $R_i (i = 1,2,\cdots,K-1)$.

Generate the jth sample $\phi_K(j)$ from the hypercube $[0,1]^K$ according to Eq. (6) -(8).

Obtain Hammersley sequence matrix $\boldsymbol{z}(j)$ by replacing K non-zero elements in the matrix $\tilde{\boldsymbol{\varepsilon}}$ with K elements of $\phi_K(j)$.

Generate the jth sample $\boldsymbol{\varepsilon}^j = \{\varepsilon_{dh}^j\}$ of the uncertain matrix $\tilde{\boldsymbol{\varepsilon}}$ by $\varepsilon_{dh}^j = \underline{\varepsilon}_{dh} + z_{dh}(j) \cdot (\overline{\varepsilon}_{dh} - \underline{\varepsilon}_{dh})$, where the subscript dh denotes the element of the dth row and the hth column.

Let $j = j+1$ and repeat steps (2)~(5) for N_{samp} times;

Use the N_{samp} samples of the uncertain matrix $\tilde{\boldsymbol{\varepsilon}}$ produced by sampling to construct expectation model approximating to problem (3) as follows.

$$\min \quad z(\boldsymbol{x}) = \boldsymbol{c}^T \boldsymbol{x} + \sum_{k=1}^{n} \sum_{j=1}^{N_{samp}} \alpha_k (d_k^{j+} + d_k^{j-}) + e_{samp} \quad (9)$$

$$s.t. \begin{cases} d_k^{j+} = [f_k(\boldsymbol{x}, \boldsymbol{\varepsilon}^j) - \bar{b}_k] \vee 0, \\ d_k^{j-} = [\underline{b}_k - f_k(\boldsymbol{x}, \boldsymbol{\varepsilon}^j)] \vee 0, \\ d_k^+, d_k^- > 0, \\ \sum_{i=1}^{m} x_i = M \\ x_i \geq 0 \\ i = 1,2,3,\cdots,\mathrm{m}; k = 1,2,3,\cdots,n. \end{cases} \quad (10)$$

where e_{samp} is a penalty term related to sampling error.

The random genetic algorithm based on HSS

Genetic algorithm (GA) is a stochastic global search technique and has been receiving increased attention because of a series of successful applications in blending processes and other different branches of engineering [13-14]. However, it is observed that the local convergence of GA is slow especially for constrained optimization problems. To improve the performance of GA, the Hammersley stochastic GA [15] is adopted to solve the stochastic optimization problem for blending operation.

The decision vector $\boldsymbol{x} = (x_1, x_2, \cdots x_m)$ is the chromosome. In crossover and mutation steps, only one variable x_i is selected as the operating variable and expressed by a binary.

Supporting that the size of population is *popsize*, the *popsize* samples are uniformly generated from the hypercube $[0,1]^m$ by using HSS technique. Thus, the lth sample can be expressed as

$\boldsymbol{\varphi}_l = (\varphi_{l1}, \varphi_{l2}, \cdots \varphi_{lm})$, $\varphi_{li} \in [0,1]$, $l = 1,2,\cdots,$ *popsize* (11)

Thus, the ith element of the lth chromosome in the initial population can be initialized as

$$x_{li} = x_{li}^{\min} + \varphi_{li}(x_{li}^{\max} - x_{li}^{\min}) \quad , \quad i = 1,2,3,\cdots,m. \quad (12)$$

where $x_{li}^{\min}$ and $x_{li}^{\max}$ are the lower bound and the upper bound of raw material proportioning, respectively.

At first, randomly generate the number of sample; the number is updating in GA evolutionary process:

If $rand(0,1) \leq 0.5$, then

$$N_{samp} = N_{samp} + 10 rand(0,1)$$

else

$$N_{samp} = N_{samp} - 10 rand(0,1)$$

Considering that the objective function in problem (10) is to solve the minimum problem, the fitness function is constructed as follows:

$$fit(\boldsymbol{x}) = e^{-z(\boldsymbol{x})} \quad (13)$$

Here,

$$z(\boldsymbol{x}) = \boldsymbol{c}^T \boldsymbol{x} + \sum_{k=1}^{n} \sum_{j=1}^{N_{samp}} \alpha_k (d_k^{j+} + d_k^{j-}) + e_{samp} \quad (14)$$

The sampling error penalty term e_{samp} can be expressed as [15]:

$$e_{samp} = \beta(t) \cdot \frac{1}{(N_{samp})^{\alpha}} \tag{15}$$

where t is the generation number of GA; α is sampling-method-related constant and taken to be 1.4; $\beta(t)$ is a weighting function where an exponential weighting function can be derived:

$$\beta(t) = \frac{\beta_0}{\eta^t} \tag{16}$$

where β_0 is a small constant which takes the value of 0.001; η is also a constant which takes the value of 0.92.

The HSS random genetic algorithm for blending operation with uncertain parameters is summarized as follows:

Step 1: Generate the initial population $P(t)$ according to Eq(11)-(12). That is, by using HSS technique, firstly take $m-1$ prime numbers to generate the uniform samples from $[0,1]^m$ as equation (11), the length of samples is $popsize$. Then, initial the chromosomes according to equation (12).

Step 2: Select the sample numbers N_{samp} by a random move mentioned in Section 4.3, and obtain the approximate expectation model by the procedure in Section 3.2.

Step 3: Evaluate $P(t)$ with the fitness function (13).

Step 4: Crossover: randomly choosing pairs of individuals from the current population with probability p_c. That is, firstly assign a random value to a temporary parameter p by HSS technique, if p is less than the crossover probability p_c, the two individuals are selected to be crossover pairs. Then randomly select an element in each individual and express them by the binaries. From the two binaries, randomly deciding a single crossover bit and exchanging the bit at the point.

Step 5: Mutation: firstly express each individual by a binary. Then change the value of each bit of the binary with probability p_{m}. That is, firstly assign a random value to a temporary parameter p by HSS technique, if p is less than the mutation probability p_{m}, the current bit of the individual is changed.

Step 6: Check the termination condition. If the evolving process satisfies the convergence condition or reaches the maximum generation predetermined, the evolution terminates and returns the best solution in current population; else loop to Step 2.

EXPERIMENTAL VALIDATIONS AND ANALYSIS

The proposed stochastic optimization method was tested by the production data acquired from a blending process of alumina smelting production, located in China. In this blending process, the composition of raw materials is not stable and has large time delay of measurements, so it is very difficult to real-time obtain the accurate values of these compositions. In order to take the parametric uncertainties into consideration, they will be regarded as random variables following the uniform distribution on the closed intervals. In the alumina blending process, there are nine kinds of raw materials including four kinds of bauxite with different grade, limestone, alkali powder, coal and two kinds of returned materials. So, $m=9$ in the stochastic optimization problem (3). Each kind of material consists of five kinds of compositions(i.e. CaO, Na_2O, SiO_2, Fe_2O_3, Al_2O_3), so $\tilde{\boldsymbol{\varepsilon}}$ is a 5×9 matrix. $\tilde{\varepsilon}_{dh}$ represents the dth component in the hth raw material, and the content can be expressed as the random number which follows the uniform distribution. The quality indices of blended product (raw slurry) mainly includes aluminum standard, alkali standard and calcium standard, namely $n=3$ in the problem(3). Supposing that $\tilde{\boldsymbol{\varepsilon}}_{1i}, \tilde{\boldsymbol{\varepsilon}}_{2i}, \tilde{\boldsymbol{\varepsilon}}_{3i}, \tilde{\boldsymbol{\varepsilon}}_{4i}, \tilde{\boldsymbol{\varepsilon}}_{5i}$ in the matrix $\tilde{\boldsymbol{\varepsilon}}$ represent the quality percentage of CaO, Na_2O, SiO_2, Fe_2O_3 in the ith raw material, respectively; $f_1(\boldsymbol{x}, \tilde{\boldsymbol{\varepsilon}})$ is the function of aluminum standard, $f_2(\boldsymbol{x}, \tilde{\boldsymbol{\varepsilon}})$ and $f_3(\boldsymbol{x}, \tilde{\boldsymbol{\varepsilon}})$ represent the function of alkali standard and calcium standard, respectively. According to the requirement of alumina blending operation, three functions can be expressed as follows:

$$f_1(\boldsymbol{x}, \tilde{\boldsymbol{\varepsilon}}) = \frac{\sum_{i=1}^{m} \tilde{\boldsymbol{\varepsilon}}_{5i} x_i}{\sum_{i=1}^{m} \tilde{\boldsymbol{\varepsilon}}_{3i} x_i} \tag{17}$$

$$f_2(\boldsymbol{x}, \tilde{\boldsymbol{\varepsilon}}) = \frac{1.654 \sum_{i=1}^{m} \tilde{\boldsymbol{\varepsilon}}_{2i} x_i}{\sum_{i=1}^{m} \tilde{\boldsymbol{\varepsilon}}_{5i} x_i + 0.6375 \sum_{i=1}^{m} \tilde{\boldsymbol{\varepsilon}}_{4i} x_i} \tag{18}$$

$$f_3(\boldsymbol{x}, \tilde{\boldsymbol{\varepsilon}}) = \frac{1.071 \sum_{i=1}^{m} \tilde{\boldsymbol{\varepsilon}}_{1i} x_i}{\sum_{i=1}^{m} \tilde{\boldsymbol{\varepsilon}}_{3i} x_i} \tag{19}$$

The production data are used to demonstrate the effectiveness of the proposed method. The experimental are shown in Figure 1 and Table 1, where the optimization results of

proposed stochastic optimization method are compared with that of deterministic optimization method [16] which is running in the alumina blending process.

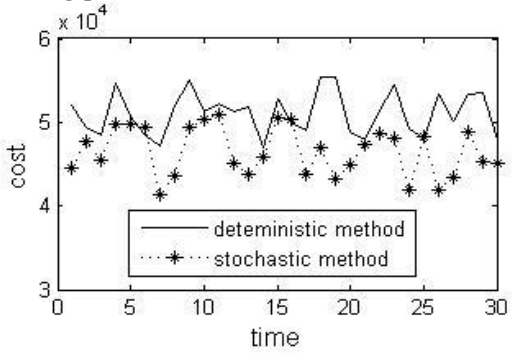

Figure 1. Comparison of raw material cost

In Figure 1, the cost of raw material acquired by stochastic method is compared with that of deterministic method. It can be seen the stochastic method can get a smaller cost, this is because that the stochastic optimization model takes into account the consumption cost of raw materials, while the objective of deterministic method is just the quality. The eligibility rate of three quality indices is quantitatively shown In Table 1. As is evident from Table 1, three quality indices is no worse than the deterministic optimization method and especially alkali standard and calcium standard are effectively improved by proposed stochastic method, which demonstrates the good robustness of proposed method. Unfortunately, the eligibility rate of aluminum standard is reduced by 0.15%. The main reason is the different optimization objective of two methods. In the deterministic method, its objective is the quality indices and aluminum standard is given more attention by weighting factor, while the objective of the stochastic method includes the cost of raw materials as well as quality indices. Thus, the stochastic method would consume the returned materials as much as possible and save the bauxites which are expensive, which can impact on the eligibility rate of aluminum standard.

Table 1. Comparison of eligibility rate of quality indices

quality index	stochastic method	deterministic method
Alumina standard f_1	92.48 %	92.63%
Alkali standard f_2	98.65 %	96.78%
Calcium standard f_3	94.38 %	90.53%

CONCLUSION

Uncertainty is an inherent characteristic in the metallurgical blending operation due to instability of raw materials, incompleteness and time delay of measurements, and unknown environmental disturbance. This paper focuses on the uncertainties in the composition parameters of raw materials and presents a stochastic optimization method based on HSS technique and GA. The proposed method takes parametric uncertainties into consideration in the stochastic optimization model, where the uncertain parameters are taken as random variables following uniform distribution, and the objective is to directly minimize the cost of raw materials and the deviation of quality indices. In order to effectively solve the blending stochastic optimization problem with multi-objective, an efficient HSS technique is utilized to transform the stochastic problem into the expectation counterpart. Furthermore, HSS technique is also used in the population initialization, crossover and mutation operation of GA to ensure the population diversity and uniformity of random operations. The HSS stochastic GA can efficiently solve the stochastic optimization problem for blending operation. The experimental results from real-world case shows the proposed stochastic optimization method not only can guarantee the quality of product but also reduce the cost of raw materials under certainty, which demonstrates a good robustness.

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