Journal of Applied Science and Engineering Innovation, Vol.4 No.1, 2017, pp.19-24
ISSN (Print): 2331-9062
ISSN (Online): 2331-9070

The Stochastic Property of Matter Motion must Disperse the Singularity of Space-Time

—Langevin equation with thermal noise; Quantum fluctuation in Planck scale and Uncertainty relation of space-time

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Abstract: For the temperature T and Planck constant h both finite, there must be the thermal and quantum fluctuation in the matter field of the gravitational collapse. Thus the motion and distribution of all microns and particles in matter field should be stochastic. By using the local Lorentz system we have written Langevin equation for the motion of each micron, and derived the uncertainty relation for microns in thermal fluctuation. According to thermal diffusion and quantum diffusion, we conclude that the position distribution of each micron (or particle) should be Gaussion function. And from the uncertainty relation for microns in thermal fluctuation, and the minimal length uncertainty relation in Planck scale, and the uncertainty relation of space-time derived by us, we have proven that the stochastic property of matter motion must disperse the singularity of space-time.

Keywords Thermal noise; Quantum fluctuation; Uncertainty of space-time; Disperse singularity

INTRODUCTION

In 1965—1970, Penrose and Hawking have proven the famous singularity theorems. The problem has been discussed by many scholars. We think the singularity theorems are only the ideas, classical and the effects of heat and quantum have been not considered. And we think the spherically symmetric (or axisymmetric) matter and metric field are the ideal mean models, and Schwarzschild's singularity and Kerr's singular ring are only the mathematical ideas.

For temperature T and Planck constant h both finite. there must be the thermal and quantum fluctuations in the matter field of the gravitational collapse. Thus the motion and distribution of all the microns (particle in chemistry) and particles in matter field should be stochastic.

We shall prove that the stochastic property of matter motion must disperse the singularity of space-time.

THE DIFFUSION PROCESS AND GAUSSIAN DISTRIBUTION OF MICRONS IN THE MATTER

We consider a matter field consisting of the microns(particle in chemistry). Our starting point is to reconsider the energy-momentum tensor of the matter distribution with velocity field:

$$T^{\mu\nu} = \rho V^{\mu} V^{\nu}, \quad (1)$$

when temperature T is finite, there must be the thermal fluctuation in the matter field. Therefore, the density ρ of the matter field should be changed. Thus we have

$$T^{\mu\nu}_{;\mu} = (\rho V^{\mu} V^{\nu})_{;\mu} = V^{\nu}(\rho V^{\mu})_{;\mu} + (\rho V^{\mu}) V^{\nu}_{;\mu}$$
$$= V^{\nu}(\rho_{,\mu} V^{\mu} + \rho V^{\mu}_{;\mu}) + (\rho V^{\mu}) V^{\nu}_{;\mu} = 0, \quad (2)$$

where $\rho_{;\mu} = \rho_{,\mu}$, since ρ is scalar. Because of the conservation of mass, thus

$$(\rho V^{\mu})_{;\mu} = \rho_{,\mu} V^{\mu} + \rho V^{\mu}_{;\mu} = 0, \quad (3)$$

That is[1]

$$\frac{d\rho}{ds} = \frac{\partial \rho}{\partial x^{\mu}} V^{\mu} = -\rho V^{\mu}_{;\mu}, \quad (4)$$

which is the continuity equation. Let $j^{\mu} = \rho V^{\mu}$ is current density, the formula (4) can be rewritten as

$$\frac{d\rho}{ds} = -(j^{\mu})_{;\mu}. \quad (5)$$

From Fick's law, we have

$$j^{\mu} = -D\rho_{;\mu}, \quad (6)$$

where D is diffusion coefficient. Inserting (6) into (5), we obtain

$$\frac{d\rho}{ds} = D(\rho^{;\mu}_{;\mu}) = D\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\mu}}\left(\sqrt{-g}\, g^{\mu\nu}\frac{\partial \rho}{\partial x^{\nu}}\right), \quad (7)$$

which is the diffusion equation in the curvilinear coordinates.

We may choose the coordinate system at any point P in any curved space-time, which allows that $\Gamma^{\sigma}_{\mu\nu}|_P = 0$. By using this local Lorentz system, it is known that the mathematical formulas of physical laws at any point P are the same as the formulas in

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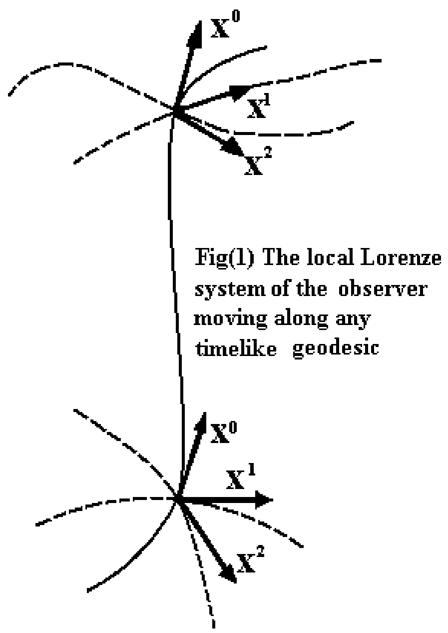
the local inertial system of Minkowski's space-time. As shown in the figure (1).



Fig(1) The local Lorenze system of the observer moving along any timelike geodesic

We consider the diffusion processes of the microns moving along respective coordinate axes x_α in the sufficiently small neighborhood around any point, therefore, the formula (7) can be rewritten as

$$\frac{d\rho}{ds} = D\frac{\partial^2 \rho}{\partial x_\alpha^2} \ , \qquad (\alpha = 1, 2, 3). \tag{8}$$

We may set $x_1 = x, x_2 = y, x_3 = z$; and s is proper time. The solutions of equations (8) are

$$\rho(x_\alpha, s) = \frac{1}{\sqrt{4\pi Ds}} e^{-x_\alpha^2/4Ds} \ , \quad (\alpha = 1,2,3) \ ,(9)$$

which show that the density distributions of the microns moving along respective coordinate axes x_α should be the Gaussian functions in the local Lorentz system x_α .

LANGEVIN EQUATION WITH THERMAL FLUCTUATION FOR THE MICRON MOTION

The diffusion process is due to the fluctuation of density, but in reality, which is due to Brownian motion. The microns in matter field can be diffused along some direction, which is just due to the Brownian motion of each micron. Therefore, we can think the diffusion process of the microns in the matter field shows that the motion of each micron can be considered as Brownian motion. The forces acting on each micron have two kinds: One is the stochastic collision force with the other microns, and another is the friction force. Thus we can derive Langevin equation for each micron.

From the fluctuation-dissipation theorem, the covariance of the stochastic collision force is

$$< F(t_1)F(t_2) >= \frac{k_B T}{\kappa}\delta(t_1 - t_2) = D\delta(t_1 - t_2), \tag{10}$$

where k_B is Boltzmann's constant, κ is friction coefficient, and D is diffusion coefficient. F(t) can be rewritten as the formal derivative of Brownian motion B(t), that is $F(t) \equiv W(t) = \frac{dB}{dt}$, since $W(t)$ have also the covariance $2D\delta(t_1 - t_2)$.

By using the local Lorentz system, we may omit the big parameter C in the Galileo's coordinates of the local inertial system, thus we may set $x_0 = t, x_1 = x, x_2 = y, x_3 = z$, and t is the coordinate time. Thus, the Langevin equations for any micron moving along three possible coordinate axes x_α should be

$$m\frac{dV_{x_\alpha}(t)}{dt} = F(t) - \beta V_{x_\alpha}(t) = W(t) - \beta V_{x_\alpha}(t) \ , \quad (\alpha = 1,2,3) \ , \tag{11}$$

which are a stochastic differential equations describing the micron motion in the local Lorentz system.. Where β is friction coefficient, where $\beta = m\xi$.

THE UNCERTAINTY RELATION FOR THE MICRON IN THERMAL FLUCTUATION

Formula (9) shows that the position variables $x_\alpha(t)$ are Gaussian stochastic processes, thus the velocity variables $V_{x_\alpha}(t)$ in Langevin equations (11) should be also Gaussian stochastic processes, since $V_{x_\alpha}(t)$ are the linear operation to $x_\alpha(t)$.

A. The position variances in curved space

The mean-square-displacement of any Brownian micron suffering thermal noise in curved space should be

$$\overline{\Delta q^\mu(t)^2} = 2\left(\frac{kT}{m\xi}\right)\Delta t = 2D_T\Delta t, \tag{12}$$

which is analogous to the Brownian particle suffering the stochastic collision of molecules, where $D_T \equiv \frac{kT}{m\xi}$, we call it the thermal-diffusional coefficient, and T is temperature.

B. Uncertainty Relation, Action Quantum and Wave-Micron Dualism for Thermal Fluctuation in curved space

1.Uncertainty Relation and Action Quantum for Thermal Fluctuation

From formula (12), we have

$$m\frac{\Delta q^{\mu}(t)}{\Delta t}\Delta q^{\mu}(t)=\left(\frac{2k}{\xi}\right)T \quad , \qquad (13a)$$

which is just

$$\Delta p^{\mu}\Delta q^{\mu}=\left(\frac{2k}{\xi}\right)T, \qquad (13b)$$

where Δp^{μ} and Δq^{μ} in Brownian motion are the Gaussian stochastic variables; and for some finite T, $\left(\frac{2k}{\xi}\right)T$ is some constant with unite: energy-second, therefore, we may define that

$$h_T\equiv\left(\frac{2k}{\xi}\right)T \quad , \qquad (14)$$

is the action quantum for the Brownian micron suffering thermal noise, which is called thermal action quantum. From formulas (13b) and (14), we obtain

$$\Delta p^{\mu}\Delta q^{\mu}=h_T \quad , \qquad (15)$$

which is the uncertainty relation for thermal fluctuation in curved space.

2. Wave-Brownian Micron Dualism for Thermal Fluctuation

From the formula (15) of the uncertainty relation for thermal fluctuation, we have

$$\Delta\lambda=\frac{h_T}{m\Delta\upsilon}, \qquad (16)$$

which is analogous to De Broglie's relation, but the formula (16) (derived by us) represents the new wave-micron dualism for the Brownian micron suffering the thermal noise in curved space.

THE DIFFUSION PROCESS AND THE MINIMAL LENGTH UNCERTAINTY RELATION OF THE PARTICLES IN STAR

A. The diffusion process of particles

There is the particle field in the process of gravitational collapse (for example, the neutron star); thus, there must be the diffusion process in quantum fluctuation.

It is known that the wave function of the particle can be considered to be the Gaussian wave packet:

$$\psi(x,t)=\frac{\sqrt{a}}{(2\pi)^{3/4}}\int_{\infty}^{\infty}e^{-\frac{a}{4}(k-k_n)^2}e^{i(kx-\omega(k)t)}dk \quad ,(17)$$

Where x represents any coordinate axis. And $\psi(x,t)^2$ shows that the probability density of the particle position is Gaussian distribution. Where (17) is the mathematical form in the local Lorentz system.

The spreading of wave packet can be explained as following: Imagine a group of particles starting at time t=o from the point x = 0, with a velocity dispersion equal to Δv. Thus, the dispersion of their positions will be Δx(t).

From the views as mentioned above, we can think the motion of the particle can be considered to be the motion of Brownian type, since the probability density of the particle position is Gaussian distribution. And these particles can diffuse along some direction, which is just due to the Brownian motion of each particle. Einstein had thought the success of probability explanation for wave mechanics means that the motion of the particle has the property of Brownian motion.

The transition probability density $f(x,t\,\|\,x_0,t_0)$ of classical microns should satisfy Fokker-Planck equation. And the "Fokker-Planck equation" associated with the probability amplitude $\psi(x,t)=<x,t\,|\,0,0>$ of microscopic particle is precisely the ordinary Schrödinger equation.

Notes on formulas (11) and (17): Langevin equation with thermal fluctuation (11) has the Brownian bridge solution, by using Brownian bridge path integral, we can also obtain Gaussian wave packer.

B. The minimal length uncertainty relation for the particle

When the space-time curvature becomes sufficiently big, there is a characteristic value. And when the component of curvature tensor in local Lorentz system satisfies the following condition

$$\Re>\Re_c=l_p^{-2} \quad , \qquad (18)$$

Where the Planck length [9]

$$l_{p\equiv}\left(\frac{Gh}{C^3}\right)^{1/2}\sim10^{-33}cm \quad , \qquad (19)$$

thus the quantum effects can be important. Considering the quantum fluctuation in Planck scale, we should use the minimal length uncertainty relation [8]

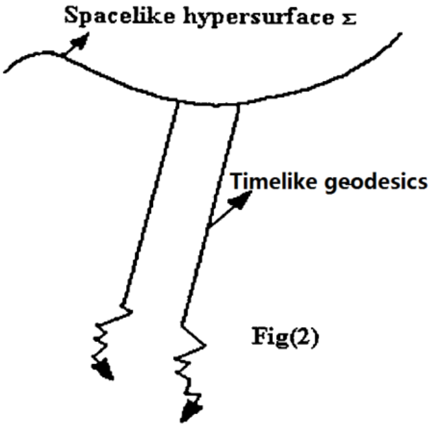
$$VxVp\geq\frac{h}{2}(1+\beta Vp^2) \quad , \qquad (20)$$

Where β is the quantity of Planck scale.

Equation (20) has appeared in the perturbation string theory, where it is implicit in the fact that strings cannot probe distances below the string scale $h\sqrt{\beta}$ .That is to say, there is a minimal length.

$$Vx_{\min} = h\sqrt{\beta} \quad , \qquad (21)$$

THE TIME-LIKE GEODESICS COULD NOT CONVERGE TO A POINT, THUS THE TIME SHOULD BE INFINITE



Fig(2)

From the minimal length uncertainty relation (20) for the microscopic particles and the uncertainty relation (15) for the microns in thermal fluctuation, we think the root-mean-square deviation Vx of the stochastic displacement along any direction could not change to zero. Therefore, all the particles or microns cannot accumulate to a point. Thus the time-like geodesics will deviate randomly from initial paths and they cannot be broken, as shown in figure(2).Obviously, there is no the congruence singularity.

On the other hand, the key equation (Raychaudhuri's equation) used in the proof of the singularity theorems is

$$\xi^c \nabla_c \theta = \frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} - R_{cd}\xi^c\xi^d \quad , \qquad (22)$$

Where θ, σ_{ab} and ω_{ab} are respectively the expansion, shear and twist of congruence. We see In (22) that the second term $-\sigma_{ab}\sigma^{ab}$ is manifestly non positive; if the congruence is hypersurface orthogonal, thus ω_{ab} =0; if Einstein's equation holds, and the strong energy condition is satisfied by T_{ab}, thus the last term is negative. Thus, we have

$$\frac{d\theta}{d\tau} + \frac{1}{3}\theta^2 \leq 0 \quad , \qquad (23)$$

Which implies

$$\frac{d}{d\tau}\theta^{-1} \geq \frac{1}{3} \quad , \qquad (24)$$

Thus

$$\theta^{-1}(\tau) \geq \theta_0^{-1} + \frac{1}{3}\tau \quad , \qquad (25)$$

Where θ_0 is the initial value of θ. Suppose that θ_0 is negative, that is to say, the congruence is initially converging. When θ goes to $-\infty$ thus

$$\tau \leq \frac{3}{|\theta_{0|}|} \quad , \qquad (26)$$

Therefore, One concluded: No past directed time-like geodesic from space-like hypersurface Σ can have length greater than $3/|\theta_0|$. The geodesics emerging normally from Σ will begin to cross by at least distance $3/|\theta_0|$. Which is the usual description of singularity theorem.

Now. according to the uncertainty relations for micron and particle[see (15) and （20）]，we think the position dispersion of each particle (or micron) could not change to zero. Thus the time-like geodesics could not converge to a point, and the nearby geodesics will deviate randomly from initial paths in a small region, thus they could not be broken, As shown in figure (2). Therefore, the time should be infinite, but time τ couldn't be equal to infinity($\tau \neq \infty$). That is to say. the time can extend infinitely. We see in (25) that $\theta(\tau) \to 0$ as $\tau \to \infty$. Thus, even if the geodesics congruence are initially converging, they cannot also converge to a point at any time. Therefore, there is no the congruence singularity.

THERMAL AND QUANTUM FLUCTUATIONS CAN DISPERSE THE SINGULARITY OF SPACE-TIME

There must be the high density micron and particle fields in process of gravitational collapse. For the temperature and Plank constant both finite, there must be the thermal and quantum fluctuations. According to the uncertainty relations (15) and (20) for microns and particles, we shall prove that the thermal and quantum fluctuations can disperse the singularity of space-time.

We think the spherically symmetric (or axisymmetric) matter field and metric field are the ideal mean models, but in reality there must be the fluctuations of matter field and metric field .We had rewritten the Einstein's field equation as stochastic differential equation [10], which shows that the stochastic matter field should determine the stochastic metric and curvature of space-time. For T and h both finite, the motion and distribution of microns and particles in star should be stochastic.

A The dispersion of Schwarzschild's singularity

After the star had formed the black hole, there are yet M.J.Q and T. Obviously, the total mass M had not changed. And we have proven that the mass density should be Gaussian distribution along any direction under the conditions of thermal and quantum fluctuations. Thus the microns and particles in star could not converge to a point in the process of the gravitational collapse. That is to say, there is no divergent density. Even if the black hole had formed, all these processes of thegravitational collapse, thermal and quantum fluctuations should be simultaneous. Considering the position dispersion Δr and velocity dispersion $\Delta \dot{r}$ of each micron (or particle) along the radial direction in local Lorentz system, both the uncertainty relations should hold. That is to say, we have

$$\mathrm{V}r\mathrm{V}P = \frac{2k}{\xi}T$$ (For microns, T $\neq$ 0), (27)

$$\mathrm{V}R\mathrm{V}P \geq \frac{\mathrm{h}}{2}(1+\beta \mathrm{V}P^2)$$ (For particles in Plank scale) , (28)

According to the Gaussian distribution of mass density and two uncertainty relations (27) and (28), we think the position dispersion could not take zero value, and there is a minimal length $\mathrm{V}r_{\min} = \mathrm{h}\sqrt{\beta}$, thus we can conclude: In Schwarzschild's solution of vacuum Einstein's equation for static, spherically symmetric space-time

$$ds^2 = -(1-2M/r)dt^2 + (1-2M/r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$, (29)

Where r could not take zero value, since the position dispersion along the radial direction is not equal to zero. Thus the uncertainty of radial position must disperse Schwarzschild's singularity.

B the dispersion of Kerr's singular ring

It is known that the singularity of Kerr's space-time appears at

$$\sum = r^2 + a^2\cos^2\theta = 0$$, (30a)

That is

$$r = 0,$$

$$\theta = \pi/2$$, (30b)

For spherical polar coordinates, $r=0$ is the origin of coordinates; for ellipsoidal coordinates, r=0 is circular dish with radius a . For ellipsoidal coordinates, $r=0$ and $\theta=\pi/2$ represent the circular ring with radius a. Thus, the singularity of Kerr's space-time is singular ring,

Now, we shall prove that the stochastic distribution and motion of the matter field in star must disperse the singular ring.

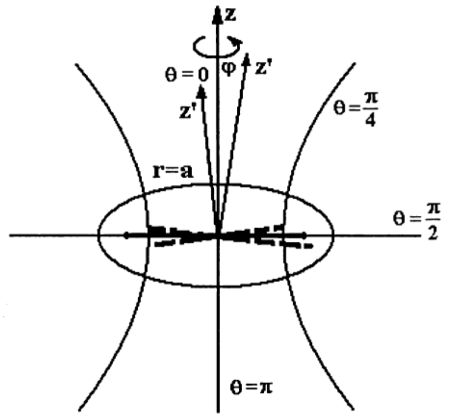
According to the uncertainty relations (15) and (20), we think r could not be fixed on the zero value in the Kerr's solution of vacuum Einstein equation and in the Schwarzschild's solution of vacuum Einstein equation, thus all the particles (or microns) in star could not accumulate on a geometric point, surface or ring.

Therefore, we can conclude that

$$\sum \equiv r^2 + a^2\cos^2\theta \neq 0$$, (31)

Since $r \neq 0$, thus $\Sigma \neq 0$, and $a = J/M \neq 0$, thus the angle θ could not be fixed on the value $\pi/2$. Therefore z = r cos $\theta \neq 0$,

For ellipsoidal coordinates, $r=0$ and $z=0$ represent the circular disk (no thickness) with radius a; but $r=0$ and $\theta=\pi/2$ represent the circular ring with radius a . Obviously, $r \neq 0$, $z \neq 0$ and $\theta \neq \pi/2$ should represent the circular dish with thickness for ellipsoidal coordinates. Because J and a are stochastic vectors, we see in (31) that r and θ should be also stochastic. Thus, we can give the conclusion: The stochastic distribution and motion of the matter field in star must disperse the Kerr's singular ring , which is indeed the circular dish with the randomly changeable thickness, radius a and the immediate axis Z', as shown in figure(3). And then (31) shows that the Kerr's solution of vacuum Einstein equation have also no space-time singularity, since we have considered the stochastic distribution and motion of the matter field in star.



Figure(3) **Ellipsoidal coordinates**

In the demonstration of this paper, considering the thermal and quantum fluctuations, the matter field could not accumulate on a geometric point or ring. Thus there is no divergent density, and then there is also no divergent metric and curvature That is to say, the stochastic property of matter motion must disperse the singularity of space-time.

UNCERTAINTY RELATION OF SPACE-TIME

According to famous Einstein's formula:

$$\mathrm{V}L = l_o\sqrt{1-\frac{v^2}{c^2}}, \mathrm{V}t = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}}, \quad (32)$$

we can obtain

$$\mathrm{V}L\mathrm{V}t = l_o t_o = \text{constant value}, \quad (33)$$

which is uncertainty relation of space-time, where $\mathrm{V}L$ and $\mathrm{V}t$ are larger than Planck scale, l_o and t_o are values at v=0, which shows v $\rightarrow 0, \mathrm{V}L = l_o, \mathrm{V}t = t_o,$ when v is near C, $\mathrm{V}L$ may be near zero, but time $\mathrm{V}t$ needs near infinite. For the curved space-time, we have

$$\mathrm{V}q = \sqrt{g_{11}}\int \delta(u)du, \mathrm{V}t = \sqrt{g_{00}}\int \delta(v)dv, \quad (34)$$

by using Schwarzschild metric in the gravitational collapse:

$$ds^2 = -\frac{1}{1-\frac{2M}{r}}dr^2 - r^2 d\theta^2 - r^2\sin^2\theta d\varphi^2 + \left(1-\frac{2M}{r}\right)dt^2, \quad (35)$$

thus, we have $\mathrm{V}q\mathrm{V}t = \sqrt{g_{11}}\sqrt{g_{00}} = 1$, (36)

this equation (36) is the uncertainty relation of space-time in the gravitational collapse, which shows that space and time are open interval, have no beginning and ending, that means there is no singularity. According to the uncertainty relations as mentioned above, we think the space-time that are larger than Planck scale (and the space-time are in gravitational collapse) could not accumulate on geometric point.

ACKNOWLEDGMENT

This work is supported by the study fund of Shandong Province (16SC176) and QTC(JG-15-1).

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