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Deriving Langevin Equation with Gravitational Noise From Stochastic Einstein Field Equation
—— Corresponding Brownian Bridge Path Integral

Lan Xin [1], Lan Chengrong [2]

[1] *Institute of biochemistry, QingDao Technical College, QingDao , China,266555*
[2] *The Department of Physics, Heze University, Heze , China 274015*

Abstract: According to the Stochastic Einstein field equation and the Langevin equation of fluctuation Path, we give the Langevin equation with gravitational noise，which describes the star-Brownian motion in stochastic curved space. By using the corresponding Brownian bridge path integral，we have derived the star wave function in stochastic curved space.

Keywords Langevin equation with gravitational noise; Brownian bridge path integral; Wave-star dualism and gravitational action quantum

INTRODUCTION

According to the Langevin equation of fluctuation path [1]， we had given the Langevin equation with quantum noise [3] ， which describes the quantum-Brownian motion of a free particle, which has the diffusion coefficient $D_Q = \frac{\hbar}{2m}$, and which is generated by the quantum fluctuation of virtual particles in vacuum. By using the corresponding Brownian bridge path integral, we obtain the wave function of a free particle.

In this paper, we give the Langevin equation with gravitational noise, which describes the star-Brownian motion in stochastic curved space, we have proven that the corresponding diffusion coefficient is $D_G = \frac{MG}{C}$; and by using the corresponding Brownian bridge path integral of a star moving in stochastic curved space, which shows that any star moving in stochastic curved space have also the wave-star dualism, which is similar to the wave-particle dualism.

I. The Langevin Equation with Gravitational Noise

It is known that the singular point as field source in non-linear Einstein field equation haven't independent motion equation .By using Bianchi identity and Einstein field equation.

$$G_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} = -8\pi k T_{\alpha\beta}, \qquad (1)$$

Go through lots of calculations, Einstein and In field [11] obtain the motion equation of the singular point as field source

$$\overset{1}{m}\ddot{\eta} = \nabla\left(\frac{\overset{1}{m}\overset{2}{m}}{r}\right), (\eta = \overset{1}{\xi}\overset{1}{m}(\tau)) \qquad (2)$$

Which is Newton motion equation only relating to the differentiation of position function $(\frac{1}{r})$, where the space coordinates $\overset{k}{\xi^m}(\tau)$ represent the position(at any time) of mass point $\overset{k}{m}(k = 1, 2, \ldots, p), r$ represents the “distance” between the coordinates $\overset{1}{\xi}s$ and $\overset{2}{\xi}s$, where $\overset{1}{\xi}s$ and $\overset{2}{\xi}s$ represent two radiuses of two infinitesimal spheres, which respectively contain mass points $\overset{1}{m}$ and $\overset{2}{m}$.We had extended Einstein field equation into stochastic Einstein field equation [9]

$$\mathfrak{R}_{\mu\nu} - \frac{1}{2}\mathfrak{G}_{\mu\nu}\mathfrak{R} = -K\mathfrak{T}_{\mu\nu}, \qquad (3)$$

Which describes that the stochastic matter source leads to the stochastic property of space-time. According to the stochastic Einstein field equation(3), considering the conservation of total energy-momentum of the mass point and the gravitational field, we have

$$-(g)(\wp^{ik} + t^{ik})_{;k} = 0, \qquad (4a)$$

all the energy-momentum tensors $\wp^{ik} = \rho v^i v^k$ and t^{ik} are the stochastic variables, we may rewrite (4a) as

$$(\rho v^i v^k)_{;k} + t^{ik}_{;k} = 0, \qquad (4b)$$

or

$$v^i(\rho v^k)_{;k} + \rho v^k v^i_{;k} = -t^{ik}_{;k}. \qquad (4c)$$

Because the conservation of mass $(\rho \equiv m\delta(r - r_0))$, the first term is equal to zero, therefore we obtain

$$\upsilon^k \upsilon^i_{;k} = -\left(\frac{1}{\rho}\right) t^{ik}_{;k} \equiv -\left(\frac{1}{\rho}\right) F^i, \qquad (5)$$

Which is the stochastic motion equation of any mass point (or any star) moving in stochastic gravitational field (or stochastic curved space), where F should be the stochastic exchange force, since

Corresponding Author: Lan Xin, Institute of biochemistry, QingDao Technical College, QingDao ,266555,China.

t^{ik} and $t^{ik}_{;k}$ are stochastic variables. And $F^i = \frac{dp^i}{dt}$ is generated by the stochastic change of the stochastic momentum p^i on the boundary between mass point and the stochastic gravitational field. Equation (5) shows that F^i must be existent, even if there is without the other outside field. Under the action of F^i a mass point moving along some geodesic line will stochastically jump to another geodesic line, and form the probability distribution. Therefore we may rewrite equation (5) into the Langevin equation with gravitational noise $P^i \equiv P_{G:}(t)$.

Equation (6) describes the star-Brownian motion in the stochastic curved space, for example, the Brownian motion of any star moving in non-regular galaxy [10]: $\frac{dq^\mu(t)}{dt} - \upsilon(q^\mu(t),t) = \frac{P_G(t)}{M}$, (6)

Where P_G plays the role of gravitational noise with the correlation function

$$\langle P_G(t)P_G(t')\rangle = \frac{m^3G}{C}\delta(t-t'). \qquad (7)$$

Equation (6) has the Brownian bridge solution as following form

$$q^\mu(t) = q_0^\mu + \upsilon t + \int \frac{P_G(s)}{M}ds, \qquad (8)$$

Where we let $q^\mu(t)$ be a lot of Brownian bridge paths, $(q_0^\mu + \upsilon t)$ be the average path, and the last term should be the path fluctuation deviating from average path, thus we have

$$\int_0^t \frac{P_G(t)}{M}ds = X_t^{q^{\mu'},q^{\mu''}}, \qquad (9)$$

We may write $X_t^{q^{\mu'},q^{\mu''}} = X_t$, thus we have

$$q^\mu(t) = \overline{q^\mu(t)} + X_t, \qquad (10)$$

where $q^\mu(t)$ should be Brownian bridge paths in stochastic curved space, which may be defined by the following formula [2]

$$X_t^{q^{\mu'},q^{\mu''}} = B_t + q^{\mu'} + \frac{t}{t_0}(q^{\mu''} - q^{\mu'} - B_{t_0}), \qquad (11)$$

Let $X_0^{q^{\mu'},q^{\mu''}} = q^{\mu'}, X_{t_0}^{q^{\mu'},q^{\mu''}} = q^{\mu''}$, and put $0 < t_1 < t_2 < \cdots < t_n \le t_0 = t_{n+1}$, when $t'=0$ and $t''=t_0$, all the Brownian bridge paths must pass through two boundary points $(0, q^{\mu'})$ and $(t_0, q^{\mu''})$ in the torsion-free case. We may suppose that $q^\mu(t)$ be stochastic geodesic lines, $\overline{q^\mu(t)}$ be average geodesic line, and X_t be fluctuation path deviating $\overline{q^\mu(t)}$ as shown in figure 1.

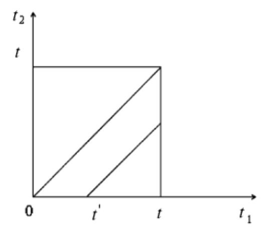

Figure 1. The integral region is equal to 2 times the size of the following triangle.

The mean-square-displacement of the star-Brownian motion in stochastic curved space should be calculated by the following method

$$\overline{(\Delta q_t^\mu)^2} = \int_0^t dt_1 \int_0^t dt_2 \upsilon(t_1)\upsilon(t_2) = \int_0^t dt_1 \int_0^t dt_2 \overline{\left(\frac{P_G(t_1)}{M}\right)\left(\frac{P_G(t_2)}{M}\right)}$$
$$= \int_0^t dt_1 \int_0^t dt_2 \left\{\frac{\left(\frac{M^3G}{C}\delta(t_1-t_2)\right)}{M^2}\right\} = \int_0^t dt_1 \int_0^t dt_2 \left\{\left(\frac{MG}{C}\right)\delta(t_1-t_2)\right\}.$$

(12)

Now, we transform the integral variables into $(t_1 - t_2) = t'$, and $0 < t_2 < t - t'$, which shows in figure2.

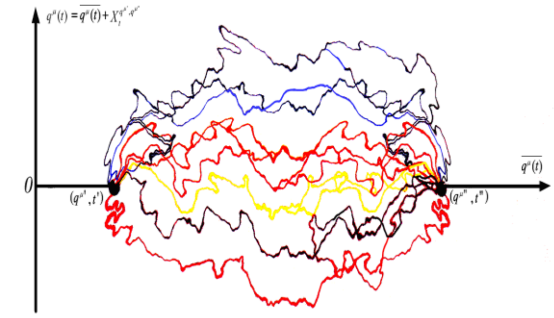

Figure 2. Where we let $q^\mu(t)$ be a lot of Brownian bridge paths, $\overline{q^\mu(t)}$ be average classical trajectory. Let $(q^{\mu'}, t')$ and $(q^{\mu''}, t'')$ be the two possible boundary points on $\overline{q^\mu(t)}$, and X_t be path fluctuations deviating from classical trajectory $\overline{q^\mu(t)}$, and these paths lie on stochastic super surfaces.

Thus we have

$$2\int_0^t dt' \int_0^{t-t'} dt_2 \left(\frac{MG}{C}\right)\delta(t') = 2\left(\frac{MG}{C}\right)(t-t')\int_0^{t'} \delta(t')dt'$$
$$= 2\left(\frac{MG}{C}\right)t\int_0^{t'} \delta(t')dt' - 2\left(\frac{MG}{C}\right)\int_0^{t'} t'\delta(t')dt' = 2\left(\frac{MG}{C}\right)t. \qquad (13)$$

In equation (13) , we have used the properties of δ -function: $\int_0^{t'} t'\delta(t')dt' = 0$, since $t'\delta(t') = 0$, and $\int_0^{t'} \delta(t')dt' = 1$. Inserting equation (13) into equation (12) , we obtain the mean-square-displacement $\overline{(\Delta q_t^\mu)^2} = 2\left(\frac{MG}{C}\right)\Delta t.$ (14)

From formula (14) , we can derive the energy fluctuation of some star in stochastic curved space $\Delta E = \frac{1}{2}Mg_{\mu\nu}\frac{\Delta q^\mu}{\Delta t}\frac{\Delta q^\nu}{\Delta t} = \frac{1}{2}Mg_{\mu\nu}\left[2\left(\frac{MG}{C}\right)\Delta t\right]\Delta t^{-2}$
$= g_{\mu\nu}\left(\frac{M^2G}{C}\right)\Delta\nu \equiv h_G\Delta\nu,$ (15)

Where $h_G = g_{\mu\nu}\left(\frac{M^2G}{C}\right)$, (16)

Which should be the action quantum for any star moving in stochastic curved space, which is called gravitational action quantum. In formula (14) , we may define the gravitation-diffusion coefficient
$D_G = \frac{MG}{C} = \frac{h_G}{g_{\mu\nu}M}$. (17a)

Comparing the quantum-diffusion coefficient derived by us [3] $D_Q = \frac{h}{2m}$, (17b) and thermaldiffusion coefficient $D=\frac{kT}{K}$, (17c) (k is Boltzmann constant, K is friction coefficient) they related to gravitational constant G Planck constant h and temperature T, they have extremely interesting physical senses.

BROWNIAN BRIDGE PATH INTEGRAL IN STOCHASTIC CURVED SPACE

Our basic postulation is that the path integral is in the stochastic curved space, and this space should be with stochastic curvature, but which is in the torsion-free case. We may suppose[4]

$$\langle q^{\mu}(t),t\,|\,q^{\mu'}(t'),t'\rangle=\prod_{n=1}^{N}\left[\int_{-\infty}^{\infty}dq_n\right]\prod_{n=1}^{N+1}k_0^{\varepsilon}(\Delta q_n), \quad (18)$$

where $K_0^{\varepsilon}(\Delta q_n)$ is the short-time amplitude, and $K_0^{\varepsilon}(\Delta q_n)=\exp\left[\left(\frac{i}{\mathrm{h}_G}\right)\frac{M}{2}\left(\frac{\Delta q_n}{\varepsilon}\right)^2\varepsilon\right], s=\frac{M}{2}\left(\frac{\Delta q_n}{\varepsilon}\right)^2\varepsilon,$ (19a) the short-time post point action $s\equiv A^{\varepsilon}(q,q-\Delta q)=\varepsilon\frac{M}{2}g_{\mu\nu}\dot{q}^{\mu}\dot{q}^{\nu}$

$$=\varepsilon\frac{M}{2}g_{\mu\nu}\left(\overline{\dot{q}^{\mu}(t)}+\dot{X}_t\right)\left(\overline{\dot{q}^{\nu}(t)}+\dot{X}_t\right)$$

$$=\varepsilon\frac{M}{2}g_{\mu\nu}\left\{\overline{\dot{q}^{\mu}(t)}\,\overline{\dot{q}^{\nu}(t)}+\overline{\dot{q}^{\mu}(t)}\dot{X}_t+\dot{X}_t\overline{\dot{q}^{\nu}(t)}+\dot{X}_t^2\right\}$$

$$=\varepsilon\frac{M}{2}\left\{\left(\frac{\Delta q_n}{\varepsilon}\right)^2+g_{\mu\nu}\left[\overline{\dot{q}^{\mu}(t)}\dot{X}_t+\dot{X}_t\overline{\dot{q}^{\nu}(t)}\right]+g_{\mu\nu}\dot{X}_t^2\right\}. \quad (19b)$$

Inserting (19b) into (19a) and (18), we obtain

$$\langle q^{\mu}(t),t\,|\,q^{\mu'}(t'),t'\rangle=\prod_{n=1}^{N}\left[\int_{-\infty}^{\infty}dq_n\right]\exp\left[\left(\frac{i}{\mathrm{h}_G}\right)\frac{M}{2}\varepsilon(g_{\mu\nu}\dot{q}^{\mu}\dot{q}^{\nu})\right]$$

$$=\prod_{n=1}^{N}\left[\int_{-\infty}^{\infty}dq_n\right]\prod_{n=1}^{N+1}\exp\left(\frac{i}{\mathrm{h}_G}\right)\frac{M}{2}\varepsilon\left\{\left(\frac{\Delta q_n}{\varepsilon}\right)^2+g_{\mu\nu}\left[\overline{\dot{q}^{\mu}(t)}\dot{X}_t+\dot{X}_t\overline{\dot{q}^{\nu}(t)}\right]+g_{\mu\nu}\dot{X}_t^2\right\}, \quad (20)$$

Where $\prod_{n=1}^{N+1}e^{\frac{iM}{2\mathrm{h}_G}\varepsilon g_{\mu\nu}\dot{X}_t^2}=\prod_{n=1}^{N+1}e^{\frac{iM}{2\mathrm{h}_G}g_{\mu\nu}\frac{dx_t^2}{dt}}=\prod_{n=1}^{N+1}e^{\frac{iM}{2\mathrm{h}_G}g_{\mu\nu}\frac{dB_t^2}{dt}}$

$$=\prod_{n=1}^{N+1}e^{\frac{iM}{2\mathrm{h}_G}g_{\mu\nu}2D_G\frac{dt}{dt}}=e^{\frac{iM}{2\mathrm{h}_G}2D_G(N+1),g_{\mu\nu}}, \quad (21a)$$

Inserting(21a)into(20),we obtain

$$\prod_{n=1}^{N+1}e^{\frac{iM}{2\mathrm{h}_G}\varepsilon g_{\mu\nu}\overline{\dot{q}^{\mu}(t)}\dot{X}_t}=\prod_{n=1}^{N+1}e^{\frac{iM}{2\mathrm{h}_G}g_{\mu\nu}\overline{\dot{q}^{\mu}(t)}\Delta X_j}, \quad (21b)$$

and $\prod_{n=1}^{N+1}e^{\frac{iM}{2\mathrm{h}_G}\varepsilon\left(\frac{\Delta q_n}{\varepsilon}\right)^2}=\prod_{n=1}^{N+1}e^{\frac{i}{\mathrm{h}_G}\left(\frac{M}{2}\varepsilon g_{\mu\nu}\overline{\dot{q}^{\mu}\dot{q}^{\nu}}\right)}.$ (21c)

Inserting (21) into (20), we obtain $\langle q^{\mu}(t),t\,|\,q^{\mu'}(t'),t'\rangle=\prod_{n=1}^{N}\left[\int_{-\infty}^{\infty}dq_n^{\mu}\right]$g

$$\prod_{j=1}^{N+1}e^{\frac{iM}{2\mathrm{h}_G}\varepsilon\left\{\overline{g_{\mu\nu}\dot{q}^{\mu}\dot{q}^{\nu}}+2g_{\mu\nu}\overline{\dot{q}^{\mu}(t)}\frac{\Delta X_j}{\varepsilon}+g_{\mu\nu}\frac{dB_t^2}{\varepsilon dt}\right\}}. \quad (22)$$

We rewrite(22)as the following form $\langle q^{\mu''}t''\,|\,q^{\mu'}t'\rangle=\int g_{q^{\mu''}|q^{\mu'}}\wp q^{\mu}(t)e^{\frac{i}{\mathrm{h}_G}S\left[q^{\mu}(t)\right]}$

$$=e^{\frac{iM}{2\mathrm{h}_G}\int_0^{t_0}\overline{g_{\mu\nu}\dot{q}^{\mu}\dot{q}^{\nu}}dt}\,e^{\frac{iM}{2\mathrm{h}_G}g_{\mu\nu}(N+1)2D_G}.$$

$$\int g_{q^{\mu''}|q^{\mu'}\left[\Delta X_1,\Delta X_2,\mathrm{L}\ \Delta X_j\right]}e^{\frac{iM}{\mathrm{h}_G}g_{\mu\nu}\overline{\dot{q}^{\mu}(t)}\lim\sum_{j=1}^{N+1}\Delta X_j}\prod_{j=1}^{N+1}d\Delta X_j, \quad (23)$$

Where the conditional Gaussian function should be written as [5]

$$g_{q^{\mu''}|q^{\mu'}}\left[\Delta X_1,\Delta X_2,\mathrm{L}\ \Delta X_j\right]=\prod_{j=1}^{N+1}\left\{\frac{e^{\frac{-\Delta X_j^2}{4\sigma_j^2}}}{\left(\sqrt{2\pi\sigma_j^2}\right)^{\frac{1}{2}}}\right\}.$$

$$\left\{\frac{e^{\frac{-(q^{\mu''}-q^{\mu'})^2}{4D_G(t''-t')}}}{\left(\sqrt{2\pi 2D_G(t''-t')}\right)^{\frac{1}{2}}}\right\}^{-1}. \quad (24)$$

In the torsion-free case, the mathematical structure of formula(23)represents the Brownian bridge path integral in stochastic curved space. This mathematical form is analogous to the calculation of the conditional expectation for the probability amplitudes along respective Brownian bridge paths $q^{\mu}(t)$ to the probability amplitude of boundary interval $(q^{\mu''}-q^{\mu'})$.[6] We integrate respectively to each independent increment $|\Delta X_j|$ of path fluctuation in (23), by using Stratonovich stochastic integral, which has usual integral method,

$$I_{\Delta X_j}=\int_{-\infty}^{\infty}\left(\frac{1}{\sqrt{2\pi\sigma_j^2}}\right)^{\frac{1}{2}}e^{\frac{-|\Delta X_j|^2}{4\sigma_j^2}}e^{\frac{iMg_{\mu\nu}^{\frac{1}{2}}\overline{\dot{q}^{\mu}(t)}}{\mathrm{h}_G}|\Delta X_j|}d\,|\Delta X_j|$$

$$=\int_{-\infty}^{\infty}\left(\frac{1}{\sqrt{2\pi\sigma_j^2}}\right)^{\frac{1}{2}}(2\sigma_j)e^{-\lambda^2\sigma_j^2}\int_{-\infty}^{\infty}e^{-(u-i\sigma_j\lambda)^2}du, \quad (25)$$

Where

$u\equiv\frac{|\Delta X_j|}{2\sigma_j},\lambda\equiv\frac{Mg_{\mu\nu}^{\frac{1}{2}}\overline{\dot{q}^{\mu}(t)}}{\mathrm{h}_G},du=\frac{d\,|\Delta X_j|}{2\sigma_j}$, thus can write $I=\int_{-\infty}^{\infty}e^{-(u-i\sigma_i\lambda)^2}du=\int_{-\infty}^{\infty}e^{-\xi^2}d\xi,$ (26)

Where $\xi\equiv u-i\sigma_j\lambda, du=d\xi,$ we have

$$I^2=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-(\xi^2+\eta^2)^2}d\xi d\eta, \quad (27a)$$

$$I^2=\int_0^{\infty}\int_0^{\pi}e^{-r^2}rdrd\theta=\pi\int_0^{\infty}e^{-r^2}dr^2=\pi, \quad (27b)$$

Which represents in polar coordinate form, therefore $I_{\Delta X_j}=\left[\frac{1}{\sqrt{2\pi\sigma_j^2}}\right]^{\frac{1}{2}}(2\sigma_j\sqrt{\pi})e^{-\lambda^2\sigma_j^2}.$ (27c)

Inserting (27c) into (23) , we obtain $\langle q^{\mu''}t''\,|\,q^{\mu'}t'\rangle=e^{\frac{iM}{2\mathrm{h}_G}\int_0^{t_0}\overline{g_{\mu\nu}\dot{q}^{\mu}\dot{q}^{\nu}}dt}\,e^{\frac{iM}{2\mathrm{h}_G}g_{\mu\nu}(N+1)2D_G}$

$$\left\{\frac{e^{\frac{-(q^{\mu''}-q^{\mu'})^2}{4D_G(t''-t')}}}{\left(\sqrt{2\pi 2D_G(t''-t')}\right)^{\frac{1}{2}}}\right\}^{-1}\prod_{j=1}^{N+1}\left[\frac{1}{\sqrt{2\pi\sigma_j^2}}\right]^{\frac{1}{2}}(2\sigma_j\sqrt{\pi})e^{-\lambda^2\sigma_j^2}.$$

(28)

According to the formula (17a), we can rewrite (28) as following form

$$\langle q^{\mu''}t''|q^{\mu'}t'\rangle = e^{\frac{iM}{2\hbar_G}\int_0^{t_0}\overline{g_{\mu\nu}\dot{q}^{\mu}\dot{q}^{\nu}}dt}\, e^{\frac{iM}{2\hbar_G}g_{\mu\nu}(N+1)2D_G}$$

$$\cdot\left\{\frac{e^{\frac{-(q^{\mu''}-q^{\mu'})^2}{4D_G(t''-t')}}}{\left(\sqrt{2\pi 2D_G(t''-t')}\right)^{\frac{1}{2}}}\right\}^{-1}\prod_{j=1}^{N+1}\left[\frac{1}{\sqrt{2\pi\sigma_j^2}}\right]^{\frac{1}{2}}(2\sigma_j\sqrt{\pi})\left\{e^{-\left(\frac{M^2\overline{g_{\mu\nu}\dot{q}^{\mu}\dot{q}^{\nu}}}{\hbar_G^2}\right)2\left(\frac{MG}{C}\right)\Delta t_j}\right\}$$

$$= e^{\frac{iM}{2\hbar_G}\int_0^{t_0}\overline{g_{\mu\nu}\dot{q}^{\mu}\dot{q}^{\nu}}dt}\, e^{\frac{iM}{\hbar_G}g_{\mu\nu}(N+1)D_G}\left\{\frac{e^{\frac{-(q^{\mu''}-q^{\mu'})^2}{4D_G(t''-t')}}}{\left(\sqrt{2\pi 2D_G(t''-t')}\right)^{\frac{1}{2}}}\right\}^{-1}$$

$$\cdot\prod_{j=1}^{N+1}\left[\frac{1}{\sqrt{2\pi\sigma_j^2}}\right]^{\frac{1}{2}}(2\sigma_j\sqrt{\pi})\left\{e^{-\left(\frac{M^2\overline{g_{\mu\nu}\dot{q}^{\mu}\dot{q}^{\nu}}}{\hbar_G^2}\right)2\left(\frac{MG}{C}\right)\Delta t_j}\right\}$$

$$=\prod_{j=1}^{N+1}\left[\frac{1}{\sqrt{2\pi\sigma_j^2}}\right]^{\frac{1}{2}}(2\sigma_j\sqrt{\pi})e^{\frac{iM}{\hbar_G}g_{\mu\nu}(N+1)D_G}\left[\frac{e^{\frac{-(q^{\mu''}-q^{\mu'})^2}{4D_G(t''-t')}}}{\left(\sqrt{2\pi 2D_G(t''-t')}\right)^{\frac{1}{2}}}\right]^{-1}$$

$$\cdot e^{\frac{iM}{2\hbar_G}\int_0^{t_0}\overline{g_{\mu\nu}\dot{q}^{\mu}\dot{q}^{\nu}}dt-\left(\frac{2M^2G}{\hbar_G^2 c}\right)2\left(\frac{M}{2}\overline{g_{\mu\nu}\dot{q}^{\mu}\dot{q}^{\nu}}\right)t}$$

$$=\prod_{j=1}^{N+1}\left[\frac{1}{\sqrt{2\pi\sigma_j^2}}\right]^{\frac{1}{2}}(2\sigma_j\sqrt{\pi})e^{\frac{iM}{\hbar_G}g_{\mu\nu}(N+1)D_G}\left[\frac{e^{\frac{-(q^{\mu''}-q^{\mu'})^2}{4D_G(t''-t')}}}{\left(\sqrt{4\pi D_G(t''-t')}\right)^{\frac{1}{2}}}\right]^{-1}$$

$$\cdot e^{\frac{iM}{2\hbar_G}\int_0^{t_0}\overline{g_{\mu\nu}\dot{q}^{\mu}\dot{q}^{\nu}}dt}\, e^{-\left(\frac{2M^2G}{\hbar_G^2 c}\right)2\overline{E}t}, \qquad (29)$$

take note of

$$e^{-\frac{1}{\hbar_G}\left(\frac{2M^2G}{\hbar_G C}\right)\overline{E}t}$$

$$=\prod_{j=1}^{N+1}e^{-\left(\frac{2M^2G}{\hbar_G^2 C}\right)\overline{E\Delta t}} = \prod_{j=1}^{N+1}e^{-\left(\frac{2M^2G}{\hbar_G^2 C}\right)\left[\frac{Mg_{\mu\upsilon}}{2}\left(\frac{\Delta q_j^{\mu}(\Delta t_j)}{\Delta t_j}\right)^2\right]\Delta t_j}$$

$$=\prod_{j=1}^{N+1}e^{-\left(\frac{2M^2G}{\hbar_G^2 C}\right)\left[\frac{Mg_{\mu\upsilon}}{2}\left(\frac{\Delta q_j^{\mu}(\Delta t_j)}{\Delta t_j}\right)^2\right]\left(\frac{\left(\frac{MG}{C}\right)}{\left(\frac{MG}{C}\right)}\right)}$$

$$=\prod_{j=1}^{N+1}e^{-\left(\frac{2M^4G^2}{\hbar_G^2 C^2}\right)g_{\mu\upsilon}\left(\frac{\overline{\Delta q_j^{\mu}(\Delta t_j)}^2}{2\left(\frac{MG}{C}\right)\Delta t_j}\right)} = \prod_{j=1}^{N+1}e^{-\left(\frac{4M^4G^2}{\hbar_G^2 C^2}\right)g_{\mu\upsilon}\frac{\overline{\Delta q_j^{\mu}(\Delta t_j)}^2}{2\sigma^2_{q_j^{\mu}(\Delta t_j)}}}.$$

(30)

Because the linear operation of Gaussian processes should be also Gaussian process [6] , thus the probability amplitudes of a star moving along paths $q^{\mu}(t), X_t$ and $\overline{q^{\mu}(t)}$ in Brownian bridge should have the same Gaussian distribution. Therefore, we may rewrite (29) as the following form

$$\langle q^{\mu''}t''|q^{\mu'}t'\rangle = \prod_{j=1}^{N+1}\left(2\sqrt{\pi}\sigma_{\overline{q^{\mu}(\Delta t_j)}}\right)\left(\frac{1}{\sqrt{2\pi\sigma^2_{\overline{q^{\mu}(\Delta t_j)}}}}\right)^{\frac{1}{2}}.$$

$$\cdot e^{-\left(\frac{4M^4G^2}{\hbar_G^2C^2}\right)g_{\mu\upsilon}\frac{\overline{\Delta q_j^{\mu}(\Delta t_j)}^2}{2\sigma^2_{\overline{q^{\mu}(\Delta t_j)}}}}\, e^{\frac{iM}{\hbar_G}g_{\mu\nu}(N+1)D_G}, \qquad (31)$$

$$\left[\frac{e^{\frac{-(q^{\mu''}-q^{\mu'})^2}{4D_G(t''-t')}}}{\left(\sqrt{4\pi D_G(t''-t')}\right)^{\frac{1}{2}}}\right]^{-1}\left\{e^{\frac{i}{2\hbar_G}\overline{p}\overline{q^{\mu}(t)}}\right\}\left\{e^{-\frac{i}{\hbar_G}\left(\frac{2M^2G}{i\hbar_G C}\right)\overline{E}t}\right\}$$

Where $e^{\frac{i}{2\hbar_G}\overline{p}\overline{q^{\mu}(t)}} = e^{\frac{i}{\hbar_G}\left(\frac{M}{2}\upsilon^2\right)t_0} = e^{-\frac{iM}{2\hbar_G}\int_0^{t_0}\overline{g_{\mu\nu}\dot{q}^{\mu}\dot{q}^{\nu}}dt}$. We will emphasize that $\sigma^2_{\overline{q^{\mu}(t)}}$ is the positional variance of a star moving in average trajectory $\overline{q^{\mu}(t)}$.According to Huygens-Fresnel principle of transmitting amplitude wave [4] $\psi(x_2,t_2)=\int_{-\infty}^{\infty}\langle x_2,t_2|x_1,t_1\rangle\psi(x_1,t_1)dx_1$, (32) we may rewrite formula (31), let $q^{\mu''}(t'')$ and $q^{\mu'}(t')$ be the variable boundary points, and $\overline{q^{\mu}(t)}$ be variable average trajectory in Brownian bridge. Thus,

$$\psi(\overline{q^{\mu}(t)})=\int_{-\infty}^{\infty}\langle\overline{q^{\mu}(t)}|(q^{\mu''}-q^{\mu'})\rangle\left[\frac{e^{\frac{-(q^{\mu''}-q^{\mu'})^2}{4D_G(t''-t')}}}{\left(\sqrt{4\pi D_G(t''-t')}\right)^{\frac{1}{2}}}\right]d(q^{\mu''}-q^{\mu'})$$

$$=\int_{|(q^{\mu''}-q^{\mu'})|}^{|(q^{\mu''}-q^{\mu'})|+\sigma_{\overline{q^{\mu}(t)}}}d|(q^{\mu''}-q^{\mu'})|\prod_{j=1}^{N+1}(2\sqrt{\pi}\sigma_{\overline{q^{\mu}(\Delta t_j)}}e^{\frac{iM}{\hbar_G}g_{\mu\nu}(N+1)D_G}\left(\frac{1}{\sqrt{2\pi\sigma^2_{\overline{q^{\mu}(\Delta t_j)}}}}\right)^{\frac{1}{2}}$$

$$\cdot e^{-\left(\frac{4M^4G^2}{\hbar_G^2C^2}\right)g_{\mu\upsilon}\frac{\overline{\Delta q_j^{\mu}(\Delta t_j)}^2}{2\sigma^2_{\overline{q^{\mu}(\Delta t_j)}}}}\left\{e^{\frac{i}{2\hbar_G}\overline{p}\overline{q^{\mu}(t)}-\frac{i}{\hbar_G}\left(\frac{M^2G}{i\hbar_G C}\right)\overline{E}t}\right\}$$

$$\cdot\left[\frac{e^{\frac{-(q^{\mu''}-q^{\mu'})^2}{4D_G(t''-t')}}}{\left(\sqrt{4\pi D_G(t''-t')}\right)^{\frac{1}{2}}}\right]^{-1}\left[\frac{e^{\frac{-(q^{\mu''}-q^{\mu'})^2}{4D_G(t''-t')}}}{\left(\sqrt{4\pi D_G(t''-t')}\right)^{\frac{1}{2}}}\right], \qquad (33)$$

which is multiplied by the marginal probability amplitude on two sides of formula (31) , and integrating for the following boundary condition [6] the probability density

$$f\left\{\left[|(q^{\mu''}-q^{\mu'})|+\sigma_{\overline{q^{\mu}_{(t)}}}\right]=\pm\infty,t\right\}=0, \qquad (34)$$

which shows that the fluctuating boundary distance $\left[|(q^{\mu''}-q^{\mu'})|+\sigma_{\overline{q^{\mu}_{(t)}}}\right]$ keep finite values. We see in (33) that the integral result on left side should be the amplitude wave function $\psi\left(\overline{q^{\mu}(t)}\right)$, and integral result on right side is $\sigma_{\overline{q^{\mu}(t)}}$. Thus, formula (33) becomes

$$\psi\left(\overline{q^{\mu}(t)}\right)==\prod_{j=1}^{N+1}\left(2\sqrt{\pi}\sigma_{\overline{q^{\mu}(\Delta t_j)}}\right)\sigma_{\overline{q^{\mu}(t)}}e^{\frac{iM}{\hbar_G}g_{\mu\nu}(N+1)D_G}$$

$$\left\{2\pi\sigma^2_{\overline{q^\mu(t)}}\right\}^{-\frac{1}{2}} e^{-\left(\frac{4M^4G^2}{h_G{}^2C^2}\right)g_{\mu\upsilon}\frac{\overline{q^\mu(t)}^2}{2\sigma^2_{\overline{q^\mu(t_j)}}}}$$

$$e^{-\frac{i}{2h_G}\bar{p}\overline{q^\mu(t)}-\frac{i}{h_G}\left(\frac{2M^2G}{ih_GC}\right)\bar{E}t} \quad (35)$$

.

From the formula of new action quantum $h_G = g_{\mu\nu}\left(\frac{M^2G}{C}\right)$, (16); and the gravitation-diffusion coefficient $D_G = \frac{MG}{C}$, (17a); and by using $t = -i\tau$, we may rewrite (35) as the following form

$$\psi\left(\overline{q^\mu(\tau)}\right) = \prod_{j=1}^{N+1}\left(2\sqrt{\pi}\sigma_{\overline{q^\mu(\Delta\tau_j)}}\right)\sigma_{\overline{q^\mu(\tau)}}e^{i(N+1)}$$

$$g\left(\frac{1}{\sqrt{2\pi\sigma^2_{\overline{q^\mu(\tau)}}}}\right)^{\frac{1}{2}} e^{-4g^{-1}_{\mu\nu}\frac{\overline{q^\mu(\tau)^2}}{2\sigma^2_{\overline{q^\mu(\tau)}}}} e^{-\frac{i}{2h_G}\bar{p}\overline{q^\mu(\tau)}+\frac{i}{h_G}2(g_{\mu\nu})^{-1}\bar{E}\tau}.$$

(36)

Formula (36) represents the modulated "plane wave function" along some average geodesic line, and the amplitude modulation factor is also Gaussian function, the peak is at $\overline{q^\mu(\tau)} = 0$, when $\sigma_{\overline{q^\mu(\tau)}} \to 0, \psi(\overline{q^\mu(\tau)})$ is $\delta-$ function wave packet [7], its width is $2\sigma_{\overline{q^\mu(\tau)}}$, which should diffuse with τ as Gaussian wave packet.

We may think the wave function (36) is star wave function in stochastic curved space, but in reality, which is the sample function with periodic property of the probability amplitude in the star-Brownian motion.

CONCLUSION

As mentioned above, we have given the Langevin equation with gravitational noise, which should be the motion equation of any star moving in some stochastic curved space, then we have calculated the variance

$$\overline{(\Delta q^\mu_t)^2} = 2(\frac{MG}{C})\Delta t, \quad (14)$$

Where $D_G = \frac{MG}{C}$ should be gravitation-diffusion coefficient, and we have derived

$$h_G = g_{\mu\nu}\left(\frac{M^2G}{C}\right), \quad (16)$$

which is called the action quantum for any star moving in stochastic curved space. By using corresponding Brownian bridge path integral, we have derived the star wave function $\psi\left(\overline{q^\mu(\tau)}\right)$ as shown in (36), which shows that any star moving in stochastic curved space have also the wave-star dualism, which is similar to the wave-particle dualism. We think the stochastic curved space should be a lot of stochastic 3-dimensional super surfaces, which satisfies the stochastic Einstein field equation expanded by us [9], and the star-Brownian motion should be along respective possible stochastic geodesic lines in respective possible 3-dimensional super surfaces. We think any star should suffer the gravitational noise $P_G(t)$, which is generated by the collective motion with some stochastic property of the other stars in galaxy. It is known that the wave function represent as the path integral form satisfy Schrödinger equation. Therefore, the star wave function (36) should satisfy a new "Schrödinger equation". Comparing Schrödinger equation

$$\frac{d}{dt}\varphi = \varepsilon\frac{d^2}{dx^2}\varphi, \left(\varepsilon = \frac{ih}{4\pi m}\right) \quad (37a)$$

And diffusion equation $\frac{d}{dt}\omega = D\frac{d^2}{dx^2}\omega$, (37b)

they are analogous. Rewriting Schrödinger equation as the following form

$$ih\frac{\partial}{\partial t}\varphi(x,t) = -\left(\frac{h^2}{2m}\right)\frac{\partial^2}{\partial x^2}\varphi(x,t), \quad (38a)$$

where $\left(\frac{h^2}{2m}\right) = h\left(\frac{h}{2m}\right) = hD_Q$, that is

$$\frac{\partial}{\partial t}\varphi(x,t) = \left(\frac{ih}{2m}\right)\frac{\partial^2}{\partial x^2}\varphi(x,t), \quad (38b)$$

thus

$$\varepsilon = \left(\frac{ih}{2m}\right) = iD_Q, \quad (38c)$$

Where D_Q is the quantum-diffusion coefficient given by us [3]. Similarly, we have the diffusion equation in star-Brownian motion

$$\frac{d}{d\tau}\omega = D_G\frac{d^2\omega}{dq^{\mu^2}}, (D_G = \frac{MG}{C}, \varepsilon = iD_G) \quad (39a)$$

Where D_G is the gravitation-diffusion coefficient, and the corresponding "Schrödinger equation" should be

$$ih_G\frac{\partial}{\partial\tau}\psi(q^\mu,\tau) = -h_GD_G\frac{\partial^2}{\partial q^{\mu^2}}\psi(q^\mu,\tau), \quad (39b)$$

Which is similar to (38a), thus which has the following form

$$\frac{\partial}{\partial\tau}\psi(q^\mu,\tau) = iD_G\frac{\partial^2}{\partial q^{\mu^2}}\psi(q^\mu,\tau), \quad (39c)$$

which is the "Schrödinger equation" satisfied by the star wave function $\psi(q^\mu,\tau)$.

In a word, we have proven that the Langevin equation with quantum noise is the motion equation of a free particle [3], and the Langevin equation with gravitational noise is the motion equation of any star moving in the stochastic curved space (for example, any star moving in the non-regular galaxy [10]),and the probability amplitudes with periodic property (wave functions) of them satisfy corresponding Schrödinger equations.

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