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Langevin Equation with Pseudo-Thermal Noise and Dissipative Noise in Non-Regular Galaxy —— Corresponding Brownian Bridge Path Integral

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Abstract: We give the Langevin equation with Pseudo-thermal noise and dissipative noise, which describes the star-Brownian motion in non-regular galaxy. By using the corresponding Brownian bridge Path integral, we have also obtained the wave function of star-Brownian motion, which is similar to the wave function of this star suffered gravitational noise in non-regular galaxy.

Keywords Langevin equation with pseudo-thermal noise; Brownian bridge path integral; star wave function

INTRODUCTION

We had derived the Langevin equation with gravitational noise from stochastic Einstein field equation[Lan. etc, 2017], which is the motion equation of any star moving in some galaxy with stochastic curved space. We had further derived the action quantum for any star moving in stochastic curved space, which is $\mathrm{h}_G = g_{\mu\upsilon}\left(\frac{M^2 G}{C}\right)$. And by using corresponding Brownian bridge path integral, we had derived the probability amplitude with periodic property-the wave function $\psi\left(\overline{q^{\mu}(t)}\right)$ of any star suffered gravitational noise, this is a quantization theory for any star moving in some stochastic curved space, which is analogous to our another quantum theory of a free particle suffered quantum noise of vaccum fluctuation.

In this paper, according to that the motion of any star around the galaxy nucleus should have the stochastic property, since it has suffered gravitational noise, and the space-time metric in any galaxy should be stochastic. But we may think the space-time metric in galaxy should be mean-square symmetric to galaxy nucleus. Thus, we may use the spherical coordinates in the symmetric and stochastic curved space to represent the action

$$S \equiv \varepsilon\left(\frac{m}{2}\right) g_{\mu\upsilon} \dot{q}^{\mu} \dot{q}^{\upsilon} \to \varepsilon\left(\frac{m}{2}\right) g_{\mu\mu}\left(\dot{q}^{\mu}(t)\right)^2$$

$$= \varepsilon\left(\frac{m}{2}\right) g_{\mu\mu}\left(\overline{\dot{q}^{\mu}(t)} + \dot{X}_t\right)^2 .$$

And in this paper, we think any star suffered gravitational noise of a lot of stars in galaxy should be analogous to any Brownian Particle suffered thermal noise of a lot of molecular-collisions, therefore, we may think any star suffered the gravitational noise in galaxy should be equivalent to any star suffered the pseudo-thermal noise in galaxy, thus we can give the Langevin equation with pseudo-thermal noise and corresponding action quantum $\left(\mathrm{h}_{T*} = \left(\frac{2k}{\xi}\right) T*\right)$ for the Brownian star suffered pseudo-thermal noise, and we have derived the star wave function corresponding to pseudo thermal noise, which has the same mathematical form as the star wave function corresponding to gravitational noise.

Obviously, we may use the corresponding Langevin equations with quantum noise, gravitational noise, thermal noise, pseudo-thermal noise and electromagnetic noise to describe unitedly corresponding respective "Brownian motions" of a free particle, star, mass point and charged particle.

By using corresponding Brownian bridge path integrals, we must obtain their probability amplitudes with periodic property (wave functions).

I. LANGEVIN EQUATION WITH PSEUDO-THERMAL NOISE AND DISSIPATIVE NOISE IN NON-REGULAR GALAXY

It is known that the motion equation of a Brownian particle is the following Langevin equation[Wu Dayou]

$$m\frac{d\upsilon}{dt} = -\xi'\upsilon + F(t), \tag{1}$$

Where ξ' is the coefficient of viscosity, and $F(t)$ is the stochastic collision force acting on a Brownian particle, which is generated by the thermal

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motion of a lot of molecules/.We may rewrite (1) as

$$m\frac{d\upsilon}{dt}=-m\xi\upsilon+F(t),\xi=\frac{\xi'}{m}, \qquad (2a)$$

or $$\frac{d\upsilon}{dt}=-\xi\upsilon+\frac{F(t)}{m}, \qquad (2b)$$

we further rewrite (2b) as

$$\frac{d}{dt}[\frac{dq^{\mu}(t)}{dt}-\upsilon(q^{\mu}(t),t)]=$$
$$-\xi[\frac{dq^{\mu}(t)}{dt}-\upsilon(q^{\mu}(t),t)]+\frac{F(t)}{m}, \qquad (2c)$$

Integrating (2c) , we obtain

$$\int dt\frac{d}{dt}[\frac{dq^{\mu}(t)}{dt}-\upsilon(q^{\mu}(t),t)]=$$
$$-\int_0^{\delta p}\xi[\frac{dq^{\mu}(t)}{dt}-\upsilon(q^{\mu}(t),t)]+\int dt\frac{dP_{T*}(t)}{mdt}, \qquad (2d)$$

where

$$F(t)=\frac{dP_{T*}(t)}{dt},\xi[\frac{dq^{\mu}(t)}{dt}-\upsilon(q^{\mu}(t),t)]=\xi\frac{dP}{m},$$

thus (2d) becomes

$$\frac{dq^{\mu}(t)}{dt}-\upsilon(q^{\mu}(t),t)=\frac{P_{T*}(t)-\xi\delta P}{m}, \qquad (3)$$

which is the Langevin equation with Pseudo-thermal noise and dissipative noise derived by us. We suppose that $(P_{T*}(t)-\delta P)$ play the roles of pseudo-thermal noise and dissipative noise, which lead to the path of a Brownian star stochastically deviate from its average trajectory. And this Langevin equation with pseudo-thermal noise and dissipative noise has just the Brownian bridge solution as the following form

$$q^{\mu}(t)=q_0^{\mu}+\upsilon t+\int_0^t\frac{(P_{T*}(t)-\xi\delta p)}{m}ds, \qquad (4)$$

we let $q^{\mu}(t)$ be a lot of Brownian bridge paths, $q_0^{\mu}+\upsilon t$ be the average trajectory, and the last term should be the path fluctuation deviating from the average trajectory, thus we have

$$\int_0^t\frac{(P_{T*}(t)-\xi\delta p)}{m}ds=X_t^{q^{\mu'},q^{\mu''}}\equiv X_t, \qquad (5a)$$

thus formula (4) becomes

$$q^{\mu}(t)=\overline{q^{\mu}(t)}+X_t. \qquad (5b)$$

we may define

$$X_t^{q^{\mu'},q^{\mu''}}=B_t+q^{\mu'}+\frac{t}{t_0}(q^{\mu''}-q^{\mu'}-B_{t_0}), \qquad (6)$$

let $X_0^{q^{\mu'},q^{\mu''}}=q^{\mu'},X_{t_0}^{q^{\mu'},q^{\mu''}}=q^{\mu''},$ and put

$$0<t_1<t_2<\cdots<t_n\le t_0=t_{n+1},$$

when $t'=0$ and $t''=t_0$, all the Brownian bridge paths must Pass through two boundary points $(0,q^{\mu'})$ and $(t_0,q^{\mu''})$ as shown in figure I:

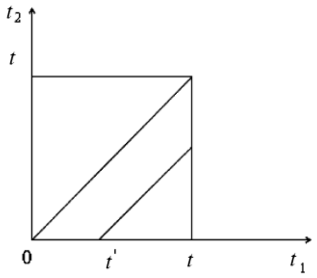

Figure 1. The integral region is equal to 2 times the size of the following triangle.

According to the fluctuation-dissipation theorem

$$<F(t_1)F(t_2)>=\frac{kT}{\kappa}\delta(t_1-t_2), \qquad (7)$$

which shows that the correlation of the stochastic gravitation in non-regular galaxy acting on some Brownian star must lead to the dissipation generated by the friction resistance, and which leads to that the path of any Brownian star stochastically deviate from its average trajectory, and as the Brownian particle in thermal motion, the mean-square-displacement of any Brownian star in the pseudo-thermal noise should be

$$\overline{\Delta q^{\mu}(t)^2}=2\left(\frac{kT^*}{m\xi}\right)\Delta t=2D_{T*}\Delta t, \qquad (8)$$

which is analogous to the Brownian particle suffering the stochastic collision of molecules,

where $D_{T*}\equiv\frac{kT*}{m\xi}$, we call it the pseudo-thermal-difussional coefficient, and T^* be defined as pseudo-temperature, which describes the stochastic degree of all stars moving in non-regular galaxy.

UNCERTAINTY RELATION, ACTION QUANTUM AND WAVE-PARTICLE DUALISM FOR THERMAL FLUCTUATION

A. Uncertainty Relation and Action Quantum for Thermal Fluctuation

From formula (8) , we have

$$m\frac{\Delta q^{\mu}(t)}{\Delta t}\Delta q^{\mu}(t)=\left(\frac{2k}{\xi}\right)T^*, \qquad (9a)$$

which is just

$$\Delta p\Delta q^{\mu}=\left(\frac{2k}{\xi}\right)T^*, \qquad (9b)$$

where Δp and Δq^{μ} in Brownian motion are the Gaussian stochastic variables; and for some finite $T^*,\left(\frac{2k}{\xi}\right)T^*$ is some constant with unite: erg-second, since k is Boltzmann constant and the equation (2a) shows that the unit of ξ should be $(\text{second})^{-1}$,therefore, we may define that

$$h_{T^*}\equiv\left(\frac{2k}{\xi}\right)T^* \qquad (10)$$

is the action quantum for the Brownian star suffering pseudo-thermal noise, which is called pseudo-thermal action quantum. From formulas (9b)and(10),we obtain

$$\Delta p \Delta q^{\mu} = \hbar_{T^*}, \quad (11)$$

which is the uncertainty relation for pseudo-thermal fluctuation.

B.Wave-Brownian Star Dualism for Pseudo-Thermal Fluctuation

From the formula (11) of the uncertainty relation for pseudo-thermal fluctuation, we have

$$\Delta\lambda = \frac{\hbar_{T^*}}{m\Delta\upsilon}, \quad (12)$$

which is analogous to De Broglie's relation, but the formula (12) (derived by us) represents the new wave-star dualism for the Brownian star suffering the pseudo-thermal noise in non-regular galaxy.

CORRESPONDING BROWNIAN BRIDGE PATH INTEGRAL

According to the Brownian bridge solution(4)and (5)of the Langevin equation with pseudo-thermal noise and dissipative noise, we can give the corresponding Brownian bridge path integral, which is just the probability amplitude with periodic property of the Brownian star suffering pseudo-thermal noise and dissipative noise.

We will give the detailed calculation process, First we suppose that the action of some Brownian star moving along any Brownian bridge path $q^{\mu}(t)$ between $(0,q^{\mu'})$ and $(t_0,q^{\mu''})$ is

$$S[q^{\mu}(t)] = S[\overline{q^{\mu}(t)}] + S[X_t]. \quad (13)$$

the total probability amplitude of a Brownian star starting from the initial point $(0,q^{\mu'})$ to end point $(t_0,q^{\mu''})$ should obtain by using the coherent superposition of the respective probability amplitudes along all the possible paths of Brownian bridge. The total probability amplitude is the transition probability amplitude of a Brownian star moving in Brownian bridge, thus we have.

$$\langle q^{\mu''}t''|q^{\mu'}t'\rangle = \int g_{q^{\mu''}|q^{\mu'}}[q^{\mu}(t)]\wp q^{\mu}(t) e^{\frac{i}{\hbar_{T^*}}S\left[q^{\mu}(t)\right]}, \quad (14)$$

the superposition coefficient $g_{q^{\mu''}|q^{\mu'}}[q^{\mu}(t)]$ should be the conditional Gaussian function. And the probability amplitude along respective bridge paths $q^{\mu}(t)$ are

$$e^{\frac{i}{\hbar_{T^*}}S[q^{\mu}(t)]} = e^{\frac{i}{\hbar_{T^*}}S[\overline{q^{\mu}(t)}+X_t]} = e^{-\frac{i}{\hbar_{T^*}}\int_0^{t_0}\frac{1}{2}mg_{\mu\mu}[\dot{\overline{q^{\mu}(t)}}+\dot{X}_t]^2dt}$$
$$= e^{\frac{i}{\hbar_{T^*}}\int_0^{t_0}\frac{1}{2}mg_{\mu\mu}[\dot{\overline{q^{\mu}(t)}}]^2dt}\, e^{\frac{i}{\hbar_{T^*}}mg_{\mu\mu}\dot{\overline{q^{\mu}(t)}}\int_0^{t_0}dX_t}\, e^{\frac{i}{\hbar_{T^*}}\int_0^{t_0}\frac{1}{2}mg_{\mu\mu}\dot{X}_t dX_t} \quad (15)$$

Differentiating(6),we have

$$\begin{cases} dx_t = dB_t + \dfrac{(q^{\mu''}-q^{\mu'}-B_{t_0})}{t_0}dt, \\ (dx_t)^2 = (dB_t)^2 + 2\dfrac{(q^{\mu''}-q^{\mu'}-B_{t_0})}{t_0}dtdB_t \\ +\left(\dfrac{(q^{\mu''}-q^{\mu'}-B_{t_0})}{t_0}\right)^2 dtdt, \end{cases} \quad (16)$$

because $dtdB_t$ =0, $dtdt = 0$, and

$$(dx_t)^2 = (dB_t)^2 = 2D_{T^*}dt, \quad (17)$$

where D_{T^*} is the pseudo-thermal-difussional coefficient. Thus, in formula (15) we may write

$$e^{\frac{im}{2\hbar_{T^*}}\int_0^{t_0}g_{\mu\mu}\frac{(dx_t)^2}{dt}} = e^{\frac{img_{\mu\mu}}{2\hbar_{T^*}}\int_0^{t_0}\frac{(dB_t)^2}{dt}}$$
$$= e^{\frac{img_{\mu\mu}}{2\hbar_{T^*}}\lim_{\Delta t\to 0}\sum_{j=1}^{n+1}2D_{T^*}\frac{\Delta t_j}{\Delta t}} = e^{\frac{img_{\mu\mu}}{2\hbar_{T^*}}(n+1)2D_{T^*}}.$$

Therefore, the Brownian bridge path integral (14) can be re-written as

$$\langle q^{\mu''}t''|q^{\mu'}t'\rangle = \int g_{q^{\mu''}|q^{\mu'}}\left[q^{\mu}(t)\right]\wp q^{\mu}(t)e^{\frac{i}{\hbar_{T^*}}S\left[q^{\mu}(t)\right]}$$
$$= e^{\frac{im}{2\hbar_{T^*}}\int_0^{t_0}g_{\mu\mu}\dot{\overline{q^{\mu}(t)}}^2dt}\, e^{\frac{img_{\mu\mu}(n+1)2D_{T^*}}{2\hbar_{T^*}}}$$
$$\int g_{q^{\mu''}|q^{\mu'}[X_t]}e^{\frac{img_{\mu\mu}\dot{\overline{q^{\mu}(t)}}}{\hbar_{T^*}}\int_0^{t_0}dX_t}\wp X_t, \quad (18)$$

which shows that the integration variable has changed from Brownian bridge path $q^{\mu}(t)$ to the path fluctuation X_t deviating from average trajectory $\overline{q^{\mu}(t)}$, and which shows in figure (2):

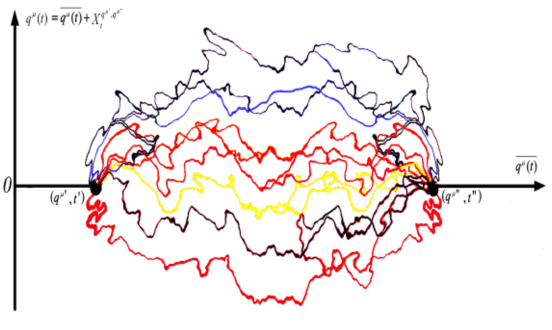

Figure 2. Where we let $q^{\mu}(t)$ be a lot of Brownian bridge paths, $\overline{q^{\mu}(t)}$ be average classical trajectory. Let $(q^{\mu'},t')$ and $(q^{\mu''},t'')$ be the two possible boundary points on $\overline{q^{\mu}(t)}$, and X_t be path fluctuations deviating from classical trajectory $\overline{q^{\mu}(t)}$, and these paths lie on stochastic super-surfaces.

In formula (18), we rewrite

$$e^{\frac{img_{\mu\mu}\overline{\dot{q}^{\mu}(t)}}{\hbar_{T^*}}\int_0^{t_0}dX_t} = e^{\frac{img_{\mu\mu}\overline{\dot{q}^{\mu}(t)}}{\hbar_{T^*}}\lim\sum_{j=1}^{n+1}(X_j - X_{j-1})} = e^{\frac{img_{\mu\mu}\overline{\dot{q}^{\mu}(t)}}{\hbar_{T^*}}\lim\sum_{j=1}^{n+1}\Delta X_j} \quad (19)$$

Inserting (19) into (18), we have

$$\langle q^{\mu''}t''|q^{\mu'}t'\rangle == e^{\frac{im}{2\hbar_{T^*}}\int_0^{t_0}g_{\mu\mu}\overline{\dot{q}^{\mu}(t)}^2 dt}\, e^{\frac{img_{\mu\mu}}{2\hbar_{T^*}}(n+1)2D_{T^*}}$$

$$\left\{\int g_{q^{\mu''}|q^{\mu'}[\Delta X_1,\Delta X_2,\mathrm{K},\Delta X_{n+1}]}\, e^{\frac{img_{\mu\mu}\overline{\dot{q}^{\mu}(t)}}{\hbar_{T^*}}\lim\sum_{j=1}^{n+1}\Delta X_j}\prod_{j=1}^{n+1}d(\Delta X_j)\right\}, \quad (20)$$

where the condition Gaussian function $g_{q^{\mu''}|q^{\mu'}}$ should be written as the following form

$$g_{q^{\mu''}|q^{\mu'}[\Delta X_1,\Delta X_2,\mathrm{K},\Delta X_{n+1}]} = \prod_{j=1}^{n+1}\left[\frac{e^{-\Delta X_j^2/4\sigma_j^2}}{\left(\sqrt{2\pi\sigma_j^2}\right)^{\frac{1}{2}}}\right]\left[\frac{e^{-\frac{(q^{\mu''}-q^{\mu'})^2}{4D_{T^*}(t''-t')}}1}{\left(\sqrt{2\pi D_{T^*}(t''-t')}\right)^{\frac{1}{2}}}\right]^{-1}. \quad (21)$$

The mathematical form of Brownian bridge path integral (18)is analogous to the calculation of the conditional expectation for the probability amplitudes along respective Brownian bridge paths $q^{\mu}(t)$ to the probability amplitude of boundary interval $(q^{\mu''}-q^{\mu'})$[1].

We integrate respectively to each independent increment ΔX_j of path fluctuation in (20), by using Strotonovich stochastic integral, which has usual integral method, we have

$$I_{\Delta X_j} = \int_{-\infty}^{\infty}\left(\frac{1}{\sqrt{2\pi\sigma_j^2}}\right)^{\frac{1}{2}} e^{\frac{-|\Delta X_j|^2}{4\sigma_j^2}}\, e^{\frac{img_{\mu\mu}^{\frac{1}{2}}\overline{\dot{q}^{\mu}(t)}}{\hbar_{T^*}}|\Delta X_j|}\, d|\Delta X_j|$$

$$=\left(\frac{1}{\sqrt{2\pi\sigma_j^2}}\right)^{\frac{1}{2}}\left\{(2\sigma_j)e^{-\lambda^2\sigma_j^2}\int_{-\infty}^{\infty}e^{-(u-i\sigma_j\lambda)^2}du\right\}, \quad (22)$$

Where

$$u \equiv \frac{|\Delta X_j|}{2\sigma_j}, \lambda \equiv \frac{mg_{\mu\mu}^{\frac{1}{2}}\overline{\dot{q}^{\mu}(t)}}{\hbar_{T^*}}, du = \frac{d|\Delta X_j|}{2\sigma_j},$$

thus we can write

$$I = \int_{-\infty}^{\infty}e^{-(u-i\sigma_i\lambda)^2}du = \int_{-\infty}^{\infty}e^{-\xi^2}d\xi, \quad (23a)$$

Where $\xi \equiv u - i\sigma_j\lambda, du = d\xi,$ thus we have

$$I^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-(\xi^2+\eta^2)^2}d\xi d\eta, \quad (23b)$$

by using polar coordinate, we obtain

$$I^2 = \int_0^{\infty}\int_0^{\pi}e^{-r^2}rdrd\theta = \pi\int_0^{\infty}e^{-r^2}dr^2 = \pi, \quad (23c)$$

therefore

$$I_{\Delta X_j} = \left[\frac{1}{\sqrt{2\pi\sigma_j^2}}\right]^{\frac{1}{2}}(2\sigma_j\sqrt{\pi})e^{-\lambda^2\sigma_j^2}. \quad (24)$$

Inserting (24) into (20), we obtain

$$\langle q^{\mu''}t''|q^{\mu'}t'\rangle = e^{\frac{im}{2\hbar_{T^*}}\int_0^{t_0}g_{\mu\mu}\overline{\dot{q}^{\mu}(t)}^2 dt}\, e^{\frac{im}{2\hbar_{T^*}}g_{\mu\mu}(n+1)2D_{T^*}}$$

$$\left\{\frac{e^{\frac{-(q^{\mu''}-q^{\mu'})^2}{4D_{T^*}(t''-t')}}}{\left(\sqrt{4\pi D_{T^*}(t''-t')}\right)^{\frac{1}{2}}}\right\}^{-1}\prod_{j=1}^{N+1}\left[\frac{1}{\sqrt{2\pi\sigma_j^2}}\right]^{\frac{1}{2}}(2\sigma_j\sqrt{\pi})e^{-\lambda^2\sigma_j^2}. \quad (25)$$

According to the formula (8) of pseudo-thermal-diffusional coefficient, we have

$$\overline{(\Delta X_j)^2} = \sigma_j^2 = 2D_{T^*}\Delta t_j = 2\left(\frac{kT^*}{m\xi}\right)\Delta t_j, \quad (26a)$$

We can rewrite (25) as the following formula

$$\langle q^{\mu''}t''|q^{\mu'}t'\rangle = e^{\frac{im}{2\hbar_{T^*}}\int_0^{t_0}g_{\mu\mu}\overline{\dot{q}^{\mu}(t)}^2 dt}\, e^{\frac{im}{\hbar_{T^*}}g_{\mu\mu}(n+1)D_{T^*}}$$

$$\left\{\frac{e^{\frac{-(q^{\mu''}-q^{\mu'})^2}{4D_{T^*}(t''-t')}}}{\left(\sqrt{4\pi D_{T^*}(t''-t')}\right)^{\frac{1}{2}}}\right\}^{-1}\prod_{j=1}^{n+1}\left[\frac{1}{\sqrt{2\pi\sigma_j^2}}\right]^{\frac{1}{2}}$$

$$(2\sigma_j\sqrt{\pi})\left\{e^{-\left[\frac{m^2g_{\mu\mu}\overline{\dot{q}^{\mu}(t)}^2}{\hbar_{T^*}^2}\right]2\left(\frac{kT^*}{m\xi}\right)\Delta t_j}\right\}, \quad (26b)$$

considering $\left(\frac{kT^*}{\xi}\right) = \frac{1}{2}\hbar_{T^*}$ in above formula (26b), we may rewrite

$$\left\{e^{-\left[\frac{m^2g_{\mu\mu}\overline{\dot{q}^{\mu}(t)}^2}{\hbar_{T^*}^2}\right]2\left(\frac{kT^*}{m\xi}\right)\Delta t_j}\right\} = \left\{e^{-\frac{1}{\hbar_{T^*}}\left(\frac{mg_{\mu\mu}\overline{\dot{q}^{\mu}(t)}^2}{2}\right)4\left(\frac{kT^*}{\hbar_{T^*}\xi}\right)(-\Delta\tau_j)}\right\}$$

$$=\left\{e^{-\frac{1}{\hbar_{T^*}}\left(\frac{mg_{\mu\mu}\overline{\dot{q}^{\mu}(t)}^2}{2}\right)2(-i\Delta\tau_j)}\right\} = \left\{e^{-\frac{i}{\hbar_{T^*}}(2\bar{E}\Delta\tau_j)}\right\}$$

$$= \left\{ e^{-\frac{i}{\hbar_{T^*}}\bar{E}\Delta\tau_j + \frac{i}{\hbar_{T^*}}\left[\frac{m g_{\mu\mu}}{2}\left(\frac{\overline{\Delta q^{\mu}(t_j)}}{\Delta t_j}\right)^2\right]\Delta\tau_j} \right\}, \tag{27a}$$

we see in (27a) that

$$e^{\frac{i}{\hbar_{T^*}}\left[\frac{m}{2}g_{\mu\mu}\left(\frac{\overline{\Delta q^{\mu}(t_j)}}{\Delta t_j}\right)^2\Delta\tau_j\right]} = e^{\frac{-1}{\hbar_{T^*}}\left[\frac{m}{2}g_{\mu\mu}\left(\frac{\overline{\Delta q^{\mu}(t_j)}}{\Delta t_j}\right)^2\Delta t_j\right]\left(\frac{\frac{kT^*}{m\xi}}{\frac{kT^*}{m\xi}}\right)}$$

$$= e^{\frac{-g_{\mu\mu}\overline{\Delta q^{\mu}(t_j)}^2}{4D_{T^*}\Delta t_j}}, \tag{27b}$$

where we have considered that $(i\tau = -t), \frac{kT^*}{\xi} = \frac{1}{2}\hbar_{T^*}$ and $D_{T^*} = \frac{kT^*}{m\xi}$.

we see in (26b) that we may rewrite

$$e^{\frac{im}{2\hbar_{T^*}}\int_0^{t_0} g_{\mu\mu}\overline{\dot{q}^{\mu}(t)}^2 dt} = e^{\frac{i}{2\hbar_{T^*}}g_{\mu\mu}\bar{p}\overline{q^{\mu}(t)}}. \tag{27c}$$

Inserting (27a, b, c) into (26b), we obtain

$$\langle q^{\mu''}t'' \,|\, q^{\mu'}t'\rangle = e^{\frac{i}{2\hbar_{T^*}}g_{\mu\mu}\bar{p}\overline{q^{\mu}(t)}}\, e^{\frac{im}{\hbar_{T^*}}g_{\mu\mu}(n+1)D_{T^*}}$$

$$\left\{\frac{e^{\frac{-(q^{\mu''}-q^{\mu'})^2}{4D_{T^*}(t''-t')}}}{\left(\sqrt{4\pi D_{T^*}(t''-t')}\right)^{\frac{1}{2}}}\right\}^{-1} \prod_{j=1}^{N+1}\left[\frac{1}{\sqrt{2\pi\sigma_j^2}}\right]^{\frac{1}{2}}(2\sigma_j\sqrt{\pi})$$

$$\left\{ e^{\frac{-g_{\mu\mu}\overline{\Delta q^{\mu}(t_j)}^2}{4D_{T^*}\Delta t_j}}\, e^{\frac{i}{\hbar_{T^*}}\bar{E}\Delta\tau_j} \right\}, \tag{28a}$$

in formula (28a) we may rewrite

$$e^{\frac{i}{\hbar_{T^*}}\bar{E}\Delta\tau_j} = e^{\frac{i}{\hbar_{T^*}}\left[\frac{m}{2}g_{\mu\mu}\left(\frac{\overline{\Delta q^{\mu}(t)}}{\Delta t_j}\right)^2\right]\Delta\tau_j}, \tag{28b}$$

considering $\overline{q^{\mu}(t)} = \sum_j \overline{\Delta q^{\mu}(t_j)}$ is the independent increment process, and $\bar{E}\tau = \bar{p}c\tau = \bar{p}\overline{q^{\mu}(\tau)}, \bar{E}t = \bar{p}ct = \bar{p}\overline{q^{\mu}(t)}$, thus

$$\bar{E}\tau = \bar{E}t. \tag{28c}$$

Therefore, (28a) becomes

$$\langle q^{\mu''}t'' \,|\, q^{\mu'}t'\rangle = \prod_{j-1}^{n+1}(2\sqrt{\pi}\sigma_j)e^{\frac{im}{\hbar_{T^*}}g_{\mu\mu}(n+1)D_{T^*}}$$

$$\left\{\frac{e^{\frac{-(q^{\mu''}-q^{\mu'})^2}{4D_{T^*}(t''-t')}}}{\left(\sqrt{4\pi D_{T^*}(t''-t')}\right)^{\frac{1}{2}}}\right\}^{-1}$$

$$\left[\frac{1}{\sqrt{2\pi\sigma_j^2}}\right]^{\frac{1}{2}} e^{\frac{-g_{\mu\mu}\overline{q^{\mu}(t)}^2}{4D_{T^*}t}} \left\{ e^{\frac{i}{2\hbar_{T^*}}g_{\mu\mu}\bar{p}\overline{q^{\mu}(t)} - \frac{i}{\hbar_{T^*}}\bar{E}t} \right\}. \tag{29}$$

According to Huygens-Fresnel principle of transmitting amplitude wave $\psi(x,t)$

$$\psi(x_2,t_2) = \int_{-\infty}^{\infty}\langle x_2,t_2 \,|\, x_1,t_1\rangle \psi(x_1,t_1)dx_1, \tag{30a}$$

we may rewrite formula (29), let $q^{\mu'}(t')$ and $q^{\mu''}(t'')$ are the variable boundary points, and $\overline{q^{\mu}(t)}$ is the variable average trajectory in Brownian bridge. thus, we have

$$\psi(\overline{q^{\mu}(t)}) = \int_{-\infty}^{\infty}\langle \overline{q^{\mu}(t)} \,|\, (q^{\mu''} - q^{\mu'})\rangle$$

$$\frac{1}{\left(\sqrt{4\pi D_{T^*}(t''-t')}\right)^{\frac{1}{2}}} e^{\frac{-(q^{\mu''}-q^{\mu'})^2}{4D_{T^*}(t''-t')}} d(q''-q')$$

$$= \prod_{j=1}^{n+1}(2\sqrt{\pi}\sigma_j)e^{\frac{im}{\hbar_{T^*}}g_{\mu\mu}(n+1)D_{T^*}\left(\frac{1}{\sqrt{2\pi\sigma_t^2}}\right)^{\frac{1}{2}}} e^{\frac{g_{\mu\mu}\overline{q^{\mu}(t)}^2}{4D_{T^*}t}}$$

$$\left\{ e^{\frac{i}{2\hbar_{T^*}}g_{\mu\mu}\bar{p}\overline{q^{\mu}(t)} - \frac{i}{\hbar_{T^*}}\bar{E}t} \right\}$$

$$\int_{(q^{\mu''}-q^{\mu'})}^{(q^{\mu''}-q^{\mu'})+\sigma_t} d(q^{\mu''}-q^{\mu'})\left[\frac{e^{\frac{-(q^{\mu''}-q^{\mu'})^2}{4D_{T^*}(t''-t')}}}{\left(\sqrt{4\pi D_{T^*}(t''-t')}\right)^{\frac{1}{2}}}\right]^{-1}$$

$$\left[\frac{e^{\frac{-(q^{\mu''}-q^{\mu'})^2}{4D_{T^*}(t''-t')}}}{\left(\sqrt{4D_{T^*}(t''-t')}\right)^{\frac{1}{2}}}\right], \tag{30b}$$

Which is multiplied by marginal probability amplitude on two sides of formula (29), and integrating for the following boundary condition [3]: the probability density

$$f\left\{\left[|(q^{\mu''} - q^{\mu'})| + \sigma_t\right] = \pm\infty, t\right\} = 0, \tag{30c}$$

Which shows that the fluctuating boundary distance $\left[|(q^{\mu''} - q^{\mu'})| + \sigma_t\right]$ keep finite values.

We see in (30b) that the integral result on left side should be the amplitude wave function $\psi\left(\overline{q^{\mu}(t)}\right)$, and the integral result on right side is σ_t. thus the formula (30b)becomes:

$$\psi\left(\overline{q^{\mu}(t)}\right)=\prod_{j=1}^{n+1}(2\sqrt{\pi}\sigma_j)\sigma_t e^{\frac{im}{h_{T^*}}g_{\mu\mu}(n+1)D_{T^*}} \quad e^{\frac{-g_{\mu\mu}\overline{(q^{\mu}(t)}^2}{4D_{T^*}t}}$$

$$\left(\frac{1}{\sqrt{2\pi\sigma_t^2}}\right)^{\frac{1}{2}}\left\{e^{\frac{i}{2h_{T^*}}g_{\mu\mu}\bar{p}\overline{q^{\mu}(t)}-\frac{i}{h_{T^*}}\bar{E}t}\right\}, \tag{31}$$

Which is just the modulated plane wave, and the amplitude modulation factor is also Gaussian function, the peak is at $\overline{q^{\mu}(t)}=0$, when $\sigma_t \to 0$, $\psi\left(\overline{q^{\mu}(t)}\right)$ is $\delta-$functional wave packet [4] ,its width is $2\sigma_t$, which should diffuse with t as the Gaussian wave packet.

CONCLUSION

In this paper, we have proven that any star suffered the gravitational noise in galaxy should be equivalent to this star suffered the pseudo-thermal noise in galaxy. And we use the corresponding Langevin equations, and corresponding Brownian bridge path integrals, we can obtain the same wave functions (the sample probability amplitudes with periodic property),

Therefore, we further think any electron moving in the atom with many electrons should be suffered a electromagnetic noise. We may use the Langevin equation with electromagnetic noise and corresponding Brownian path bridge integral to calculate its wave function. Thus, we can give the unified quantum theory of the gravitational, electromagnetic and thermal interactions.

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