

From the Semantic Explanation of Material Implication to the Theoretical Construction of the Impossible World

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Abstract: With the aim of thoroughly explaining the lack of preciseness in natural language and the theoretical problems stemmed from paradox of material implication, the semantic explanation of material implication needs to be expanded and developed. Whether it is for the objects depicted by material implication (i.e. the formal ontology), or for the implementer of the depiction (i.e. a person's perception), the concepts related to possible-world semantics are viewed as the necessary rationale to systematically interpret implications, so as to obtain a relatively well-developed theoretical framework. Moreover, the multifaceted value in theory improvement is expected to be vital to implication, possible-world semantics, ontology, epistemology, and theory of meaning.

Keywords Material implication, Possible world, Formal logic, Semantics, Ontology

INTRODUCTION

Paradox of material implication is one of the key issues in fundamental theory of logic. Its significance is not exactly concerning about how crucial or important it is for the theoretical development and advancement regarding the discussion on modern philosophical logic issues, but is related with its theoretical research status and value of academic inquiry when the core concept of logic, "implication", has been explored from the perspectives of logic and philosophy. The interpretation and understanding of the concept of implication is determined by its semantic explanation as words in a language, and symbols in a system.

THEORETICAL SIGNIFICANCE OF THE SEMANTIC DISCUSSION ON MATERIAL IMPLICATION

First of all, it has been commonly recognized that discussions on the fundamental concepts could not be carried out unless in-depth understanding of logic itself has been obtained. The reason is, the meaning of a symbol in a system has been manifested with continuous enriching system, similar as it is, in any language system, including natural language, all the concrete regulations on the sense of a word come from the system itself. That is to say, when the semantic discussion of these concepts are conducted, the sense of a word has to be regulated by the system itself. To simplify this, an example could be the discussion on logic has not really started if it just touches upon initial symbol, formation rule and so forth. Nevertheless, even for the discipline of logic which lays emphasis on the system itself, the semantic discussion on the fundamental concepts is also of crucial importance. In the history of human thought, the development, evolution, enrichment or even reinterpretation of any concept in philosophy, natural science or other disciplines have all taken place on the semantic level, or could be concluded on the semantic

level. Since this semantic conclusion usually becomes the symbol of completion of a certain periodical development in human thought, and is normally considered as a cultural node or a formal summary of the spirit of the age. Its influence on the course of human civilization is significant and decisive. Thus, with the systematic and comprehensive investigation on the fundamental theories and fundamental technology in logic, further probing into the fundamental concepts is necessary and deserves attention, for instance, what is the meaning of system, logical connectors, (material) implication, and so forth.

SEVERAL COUNTEREXAMPLES TO THE PARADOX OF MATERIAL IMPLICATION

No matter it is when giving fundamental instruction on mathematical logic to beginners, or constructing the ingenious logical system, or probing into the ultimate question like "what is logic", the paradox of material implication is always the issue that scholars in the discipline of logic could not avoid. Specifically speaking, there are some classic sentence patterns and semantic counterexamples. These are mainly related with propositional forms like $p \rightarrow (q \rightarrow p)$ and $\neg p \rightarrow (p \rightarrow q)$, and sentence patterns like $(p \rightarrow (q \vee r)) \rightarrow ((p \rightarrow q) \vee (p \rightarrow r))$.

The common counterexamples to paradox are completely irrelevant antecedent and consequent sentences to make the paradoxical feature prominent. These examples at the same time give impetus to the emergence and development of relevant implication, not tired in words here. Besides, one point worth noting is that there are some counterexamples to the above-mentioned patterns, in which the antecedent and consequent are not completely irrelevant, but truly paradoxical, making the astonishing effect. For instance, A says, "Although it was a sunny day yesterday, I am certain that if it rained yesterday, it would not be wet on the ground." "You are wrong," B refutes, "If it rains, it must be wet on the ground no matter when." A smiles and says, "I also agree what

you have said, but my statement was not wrong at all!”

Zhang San went to look for Li Si, and when he came to the crossroad, he did not know which way to go. He randomly chose to turn right, and found Li Si immediately. He said, “If I had turned left at that time, I would not see you.” Li Si answered, “exactly!” However, Zhang San said, “But if I had turned left, I would also find you.” Li Si looked at him blankly, because he could not envision how he could be found if Zhang had turned left, but this proposition is logically true.

In a scholarly presentation in Physics, the presenter has seriously declared “If Galileo had used a lead ball instead of an iron ball, two balls would not land at the same time, which obeys Aristotle’s initial assertion regarding free fall.” However, in the academic community of Physics, it has become a consensus that the results of experiments remain the same regardless of the metallic components of the ball - no matter it is a gold, silver, iron, or a lead ball. So, why this classic theory in Physics, the law of free fall, is becoming invalid in front of this seemingly true logic proposition? (A false premise makes the statement a true one.)

At another academic conference on Meteorology, a meteorologist has been presenting his propositions assertively, “If the precipitation in New York city reached 3mm yesterday, obviously this would lead to macro-precipitation in Xinjiang area in China. And the rainfall will last for one year, and it will radically change the climate conditions and hydrological environment in Taklamakan Desert, or even the whole central Asia region.” This statement seems like a fallacy, but if it were not raining in New York that day, or if the precipitation had not reached 3mm, this sentence becomes a true one.

When a civilian shouts to the king, “Your Majesty, if I were not here today, you would suffer a tragic death. And the killer would be me.” He uses the fact that he comes today (to refute the assumption of not coming today) to ensure the whole sentence is true.

Logically speaking, these statements are uncontested truth, and the antecedent and consequent are relevant. Although there could be the most rigorous logical proof, it is not difficult to discover that not all the truths are pleasant to be received, and the key issue is that the truthfulness of those statements in natural language is still controversial.

The induction of the identified false statements is all based on the construct of $\neg p \vdash p \rightarrow q$. One noteworthy feature is the statements symbolized by p and q are not irrelevant, but it is necessarily true ($p \rightarrow \neg q$), i.e. $\Box(p \rightarrow \neg q)$. With the analysis, the fundamental reason for disliking of the statements is not about determining the truthfulness of $p \rightarrow q$ or $p \rightarrow \neg q$. The real problem lies on the relationship between p and q in $\Box(p \rightarrow \neg q)$. In conclusion, people

tend not to accept any assertions in forms of $\Box(p \rightarrow \neg q)$, $\neg p \vdash p \rightarrow q$.

In addition, another type of inconceivable paradox is the semantic explanation of the negation of material implication $\neg(p \rightarrow q) \vdash p \wedge \neg q$, thus, there is: the statement “If it rains, it will not be wet on the ground” is false. (It is not true that if it rains tomorrow, it will not be wet on the ground.) This statement seems to make sense, but in fact, it is an assertion that the matter of raining tomorrow is a fact (the preceding part is true but the latter part is false). So, is it certain that it is going to rain tomorrow, logic suggests, it is.

“If Galileo had used a lead ball instead of an iron ball, two balls would not land at the same time”, this statement is false. (It is not true that if Galileo had used a lead ball instead of an iron ball, two balls would not land at the same time.) The true sentence could absurdly deduce that Galileo used a lead ball.

When a civilian said to the king, “If I were not here today, you would suffer a tragic death.” All the present civil and military officials stamped with rage and snapped at him, “Nonsense!” The civilian replied, “Well, if you say so, you must have assumed I had not been here.” Then, he went swaggeringly out of the palace.

Similarly, this type of paradox is due to the unique relationship between p and q , i.e. $\Box(p \rightarrow \neg q)$. People customarily tend to accept any assertions in the form of $\Box(p \rightarrow \neg q) \vdash \neg(p \rightarrow q)$, p .

Based on logic, these two above-mentioned inclinations should be wrong. But the problem is, in what way it could be explained with logic. Other than identifying the relationship between natural language or thought and the existing logic, including which precedes the other (which is fundamental), the more crucial issue is exploring the possibility to unify the three in logic research.

IMPLICATION AND THE POSSIBLE WORLD

With the aim of clarifying the role “ $\Box(p \rightarrow \neg q)$ ” plays in the above-mentioned situation, it is necessary to use possible-world semantics to conduct a thorough semantic analysis of material implication.

In fact, there is always a misconception when people attempt to have an analytical judgement on implication. The person who judges may customarily presuppose the possible world in which the antecedent takes place. That is to say, he assumes $p \rightarrow q$ is equivalent to $p \vdash q$. The reason is, any effective natural deduction could be converted into a tautological implication. The consistency of deduction is the essential logic attribute of implication. However, there would be a theoretical misconception caused by this conversion, and that is the essential distinction between $p \rightarrow q$ and $p \vdash q$, to simplify this, the latter affirms the truthfulness of p under certain condition, but the former does not. The former is a sentence

which puts forward a judgement that if p is true, q is also true. But the latter suggests the relationship between sentence p and sentence q , and its semantic explanation could be: in the possible world in which p is true, then q is true. The relationship between the Axiom System and the Natural Deduction, and the difference between logical proof and logical deduction are addressed here, and they are significant in theory, not tired in words here. Natural deduction could make theoretical presupposition of the premise p .

Based on the understanding, let us examine the second situation, " $\Box(p \rightarrow \neg q) \vdash \neg(p \rightarrow q)$ ". $\Box(p \rightarrow \neg q)$ indicates that in every possible world, if p is true, $\neg q$ is true, so for the analysis of $p \rightarrow q$, once there is a presupposition of p , according to the above formula, $\neg q$ is logically true. Thus, if in every possible world where every p is true, and the antecedent p is true and the consequent q is false, then in every possible world in this circumstance, $p \rightarrow q$ is false, so people tend to intuitively believe in $\Box \neg(p \rightarrow q)$. Although logically speaking, we do not investigate in the possible world where $\neg p$ is true, whether $\neg(p \rightarrow q)$ is always the case. The reason why we customarily exclude the $\neg p$ is mainly related with people's ideological rejection to $p \wedge \neg p$, or the rejection from the possible world itself. So the analysis reflects on the impreciseness of intuitive judgement. At the same time, the reason for affirming $\Box(p \rightarrow \neg q) \vdash \neg(p \rightarrow q)$ and p has been spotted.

As for the first situation, the analysis of " $\Box(p \rightarrow \neg q), \neg p \vdash p \rightarrow q$ ", when people give a true value to " $p \rightarrow q$ ", they tend to enter all the possible worlds where p is true due to the misconception of $p \vdash q$. In this case, the conclusion of $\neg q$ is valid in every possible world under the same circumstance. So the above conclusion has been drawn. Different from the second situation, $\neg p$ is also a presupposition and the antecedent in logic, but why it has been easily neglected?

The reason might be, when people see "if p is so...", the former $\neg p$ is automatically excluded because it contradicts with the antecedent, so as to enter naturally into the possible world where the prerequisite is p is true. To exemplify this, "It did not rain yesterday, but if it had rained, it would not be wet on the ground." If put in a more precise way in natural language, "It did not rain yesterday, but if in a possible world it did rain yesterday, it was not wet at all on the ground." In this case, $p \rightarrow q$ is about another possible world but not about the real world in which it did not rain yesterday. Thus, after the restatement in a more precise way, this statement is indeed false.

This also explains the theoretical significance and value of possible world in ontology, and the dependency of material implication on possible world theory. In essence, possible world not only originates from the semantic explanation of Model Logic, but also from natural language itself.

In this sense, the sentence pattern in original proposition is supposed to be " $\Box(p \rightarrow \neg q), p \vdash q$ " but not " $\Box(p \rightarrow \neg q), \neg p \vdash p \rightarrow q$ ". However, in the original case, the ones who know how to use material implication paradoxes (hereinafter referred to as paradox proposer, as opposed to "people") seem fairly confident, because the sentence pattern they assert, " $\Box(p \rightarrow \neg q), \neg p \vdash p \rightarrow q$ " (the primitive logical characterization) could be the feasible sentence pattern. Accordingly, the ambiguity of the original sentence lead to two different interpretations, or two types of logical characterization. So in the original case, different parties hold different positions.

THE CONSTRUCT OF THE IMPOSSIBLE WORLD

The problem continues, are the assertions held by paradox proposers in the original case logical? Words would not be tired here, because this is the classic contents of material implication paradox. To be specific, when $\neg p \wedge p$ acts as the antecedent of implication, all the implication propositions will be true. Besides, the strangeness of this paradox lies in that in the condition of being of true value, the propositions implied by the antecedent of the paradox could encompass totally irrelevant propositions, and other contradictory propositions, and they cover all the irrelevant and all the contradictory propositions.

Nevertheless, according to the logical rules on possible world, if and only if one possible world does not have logical contradiction, it could be called a possible world. Or we could say that any possible world itself is a collection of consistent sentences. The contradiction only exists in segment of sentence in a possible world. But what are these type of sentences attempting to describe? Nothing. The reason is a contradictory concept could not exist in any possible world, and object subordinate to it is not a part of any possible world. In this case, it becomes an empty shell without any meaning and any object subordinate to it.

Not existing yet being described, this judgement of contradiction itself is an non-existing and self-regulated "contradiction". Obviously, this is absurd.

Furthermore, the author is convinced that the world itself is a series of sentences (sentences on a generalized level, including sentence pattern and so forth), which are interwoven into countless diversified possible worlds with consistency. This is defined from the perspective of ontological level of existence. However, another indisputable fact is a contradictory sentence is a sentence, but does not belong to any possible world.

The key problem might be people tend not to recognize contradiction merely as a futile concept which does not exist in any possible world. It is without reference as a word, and as an inconsistent statement, it is without sense.

It is hardly accepted logically by people. So what goes wrong? Suppose the contradiction is not a futile concept, it must be existing outside all of the possible worlds. So the problem is where is the contradiction?

The questions being brought up indicate that other than the countless possible worlds, there must be one impossible world which contains all the contradictions. Although it is rather abstract in a conceptual and linguistic dimension, the existence is surely objective. As for any sentence or sentence pattern which contains $p \wedge \neg p$ or word contradictions, it becomes a description of the impossible world itself. Due to lack of understanding of described objects, paradoxical statements come into being.

Then, if we view a material implication paradox as a description of the contradictions in the impossible world through a sentence in a possible world, the issue of strangeness of a paradox could be resolved. According to the original contents in the paradox: under the condition of the whole sentence with true value, the propositions that contradictory antecedent could imply encompass totally irrelevant propositions, and other contradictory propositions, and they cover all the irrelevant and all the contradictory propositions. This sentence could be interpreted into: in a world where contradictions exist (it is the impossible world), all the propositions (including contradictory propositions) are valid, i.e. $p \wedge \neg p \vdash \Gamma$. (Γ is the set of propositions in the impossible world, and is to be defined hereinafter.) In this way, the strangeness of paradox could be reduced thoroughly.

The impossible world (or a contradictory world) could be described as this: the world in constructed by all the contradictory sentences, in other words, by all the sentences in the form of $p \wedge \neg p$, (sentence patterns are included). Moreover, further descriptions are as follows:

<1> Definition:

Let Δ as any formula set, Δ as a collection of all the unsatisfiable formula set (Γ and Δ can be infinite set), if and only if Γ is the union of all the components in Δ , we can say Δ' (one component in Δ) falls into the impossible world Γ . If we put the premise set Γ as a conjunctive normal form (disjunctive normal form), it will be $p \wedge \neg p \wedge p' \wedge \neg p' \wedge p'' \wedge \neg p'' \wedge \dots$

<2> Several definite descriptions:

1. $p \wedge \neg p \vdash \Gamma$
2. The impossible world is a world which contains all the propositions (including the negative propositions).
3. The impossible world is a world in which all the propositions are valid, also is a world in which all the propositions are invalid at the same time.
4. Any proposition in the impossible world is true and false at the same time.
5. Generally speaking, the impossible world is a world which consists of all the impossible things, or a contradictory world.

6. Unlike the large volume of possible worlds, there is just one impossible world. The world encompass countless possible worlds and the only one impossible world.

7. The impossible world exists outside of all the possible worlds and connects with all the possible worlds.

8. This interconnection is manifested in the following situation, suppose a contradiction (an inconsistency) emerges in a calculus system, the whole system might fall into the impossible world, and if the contradiction gets resolved or deleted, it will be led into the possible world again.

9. The difference between the possible world and the impossible world is there should be at least a slim possibility of truth in a possible world, but in the impossible world, there is no such possibility.

10. When we say a contradictory statement is false, it exists in a possible world (and could exist in any possible world), but when we say it is of true value, it exists in the impossible world.

11. When we say "false proposition is implied in all propositions ($\neg p \rightarrow (p \rightarrow q)$), we mean if the assumed false proposition is true, the whole system will fall into the impossible world, then all the propositions will be true. This is the resolution of the "strangeness" of material implication paradox.

12. The proposition $(\neg p \wedge p) \rightarrow q$ is not going to fall into the impossible world because of encompassing contradictions, because it does not assert the antecedent $(\neg p \wedge p)$ is true. But $(\neg p \wedge p) \vdash q$ falls into the impossible world because of its assertion on contradictions.

13. Any tautological and contingent indeterminable sentences are in the impossible world if and only if contradiction emerges in the same language system.

14. Adding one contradictory sentence with true value into a purely systematic possible world will convert it into one part of the impossible world. All the similar parts together are integrated into the impossible world which is purely systematic.

15. The impossible world is a world which consists of pure propositions that could not be thoroughly explained semantically by researchers. The world is an inconceivable world, so it is beyond people's perceptual knowledge. Thus, the impossible world could only be defined on the grammatical level.

16. The impossible world is an inconceivable world but not a completely untouchable world. Paradoxes and Godel's Incompleteness Theorem can reflect our understanding of the impossible world.

17. In Godel's Incompleteness Theorem, if the whole truth is put into the same possible world (system), it will fall into the impossible world, in other words, the truth can only be existing in different possible worlds.

18. Paradoxes are in the impossible world, and are the most sufficient semantic explanation we could get from the impossible world.

CONCLUSION

Someone asked Bertrand Russell how to explain the situation when the statement “If $8=1+4$, Russell and another person is the same person” is true. Russell thought about it and said, “ $8=5$, if we subtract 2 on both sides, we get $6=3$, if both sides are divided by 3, we get $2=1$. So the two persons, Russell and the other person, are actually one person.”

In fact, although the impossible world could not be thoroughly interpreted by human rationality because it is beyond people’s perceptual knowledge, the logic description could be obtained. And this is the very

reason for the “truthfulness” and “strangeness” of material implication paradox itself. In this sense, implication and possible worlds, even the sense of logic itself, are being regulated relatively more thoroughly.

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