Research on Pricing of Carbon Options Based on GARCH and B-S Model

Zhibin Liu, Shan Huang

Department of Economics and Management, North China Electric Power University, No. 619, Yonghua North Main Street, Baoding 071003, China
Corresponding author. E-mail address: liuzhibin771112@126.com (Zhibin Liu).

Abstract*:* As a carbon financial derivative, carbon options play an important role in improving the price discovery function of the carbon emission market and avoiding trading risks in the carbon emission market. In order to compete for China's pricing power in the world carbon trading market, it is necessary to study the pricing of carbon options. This paper combines the GARCH model with the B-S option pricing model to compensate for the fixed volatility of the traditional B/S option pricing model. Through the pricing study of the daily closing price of EU emission allowance futures options, the EUA's forecast price for the next 20 days is obtained, and compared with the actual price, and finally the conclusion is reached. I hope this paper can provide reference for the scientific pricing of China's upcoming carbon options trading.

Keywords Carbon option; Carbon emission rights; GARCH model; B-S model

INTRODUCTION

The global warming caused by human-induced greenhouse gas emissions has become the focus of attention of all countries in the world. In order to curb the trend of climate warming, low-carbon development has become the preferred development model for sustainable social development, and countries have taken corresponding measures [Wang Qiuzhen, 2015]. In 1990, the United States took the lead in proposing quota restrictions on carbon emissions in the Clean Air Act amendment. This is the first time in the world to introduce regulations on controlling emissions. Subsequently, the United Nations Framework Convention on Climate Change adopted in 1992 and the Tokyo Protocol in 1997 first identified carbon emission rights as a tradeable commodity, and established a carbon emission trading market [Chen Ting, 2012].

Although China does not undertake mandatory emission reduction obligations, as the world's largest emitter of greenhouse gases, energy conservation and emission reduction have both moral and practical significance. The "Twelfth Five-Year Plan" adopted in 2011 pointed out that "by 2020, the carbon intensity will be 40% to 45% lower than in 2005." In 2008, China established a carbon emission trading pilot area in Beijing, Tianjin and Shanghai, and gradually increased the number of trading pilots. By 2016, nine provinces and municipalities have become pilots. In December 2017, the National Development and Reform Commission issued the "National Carbon Emissions Trading Market Construction Plan (Power Generation Industry)", which marked the completion of the overall design of the national carbon market and officially launched. As the world's most promising carbon emission reduction market and the largest supplier of CDM projects, China is at the bottom of the entire carbon trading industry chain. Since the markets and standards of carbon trading are both abroad, China's huge emission reductions created for the global carbon market have been packaged and developed into higher-priced financial products for trading abroad after being purchased at low prices by developed countries. Therefore, only by forming and perfecting the domestic carbon trading market can we effectively compete for pricing power and discourse power in the carbon trading market and gain a favorable position in the development of the carbon trading market.

Carbon emission rights, as an asset [Wu Hongjie, 2015], to achieve marketization in the transaction, the most fundamental task at present is to conduct scientific and reasonable pricing. After long-term development, the international carbon emission trading system has formed a relatively mature spot price formation mechanism, but there is currently no research on the formation mechanism of carbon financial derivatives prices. From the development experience of the global carbon market, the carbon financial derivatives market and the development of the carbon spot market complement each other [Shu Xin, Deng Xiaowei, 2018]. The imperfect development of carbon financial derivatives will inevitably limit the development of the carbon trading market. As an indispensable derivative of the carbon trading market, carbon options play an important role in improving the price discovery function of the carbon emission market and avoiding

trading risks in the carbon emission market. However, there are still few studies on the pricing of carbon options. Therefore, the future development of the carbon trading market should focus on coordinating the development of derivatives such as carbon options, in order to achieve the steady development of the entire carbon trading market.

A carbon option is the right to sell or purchase a greenhouse gas emission right indicator at a certain price at a certain time in the future or at a certain time. The current major international carbon options include: EU Emissions Quota Futures Options (EUA), Certified Emission Reductions Futures Options (CERs) derived from the CDM, and EUR futures bullish or put options derived from the JI mechanism [Yang Jiachen , 2009], these options are all carbon futures options, a carbon financial derivative generated on the basis of carbon futures.

There are many research results on the necessity of carbon options for the current international emissions trading market. First, carbon options, as carbon financial derivatives, can hedge the uncertainty of future carbon allowances while completing emission reduction targets [MarcS. Luca,2008], while stimulating companies to increase investment in emission reduction technologies, adding more flexibility to emissions trading risk management [Valerie M. Thomas, 2016]. Secondly, by using the equilibrium model to analyze the members of the EU ETS, it can be found that the increase in the price of the emission allowance has a stronger negative impact on the newly joined member states, and the price of the carbon allowance can be reduced by introducing a carbon tax. , thereby increasing the welfare of the buyer [Corjan Brink, 2016]. Then, the reason why the price of carbon emission rights in China's CDM projects is seriously low is that China's current carbon financial transactions are all spot markets. The establishment and development of the carbon option market will help China to grasp the corresponding pricing power[Huang Ping, Wang Yulu, 2010]. Finally, enterprises determine the price of carbon emission rights through emission reductions imported from abroad, rather than their own carbon emission allowances. Using options to circumvent price risk can impose restrictions on imported carbon emissions [Ivan Diaz-Rainey and Daniel J. Tulloch, 2018].

The real option pricing model includes B-S option pricing model, binary tree pricing model and Monte Carlo simulation pricing model. The B-S option pricing model is a special case of binary tree pricing model. Most of the current research on carbon option pricing uses the B-S option pricing model for pricing. There are many advantages to establishing a BS option pricing model, such as allowing the emission reduction companies to unload the horrific psychology of high transaction costs and fear of losing liquidity and causing poor financial flow. This advantage is better than auctions to win control

companies. Welcome [Shi Shengwei, Huang Tongcheng, 2005]. Moreover, by comparing the three methods of real option pricing, namely B-S option pricing model, Monte Carlo simulation method and binary tree method, it can be concluded that the B-S option model is more suitable for carbon trading pricing [Zhu Yuezhao, Chen Hongxi, Zhao Zhimin, 2012]. Of course, some scholars have questioned the practicality of the B-S option pricing model. The carbon option price yield shows a sharp fat tail distribution, conditional heteroscedasticity and fractalism. The traditional B-S option pricing model cannot make accurate pricing [Zhang Chen, 2015]. The advantage of artificial neural networks is that there are no subjective assumptions about the market, which is not available in the B-S option pricing model [Liu Xubin, 2011]. In this regard, many scholars have improved the B-S option pricing model. The transaction cost variable is added to the B-S option pricing model, and it can be compensated for its shortcomings due to the constant price volatility of carbon emission rights in a short period of time [Zhao Xiaopan, 2016]. Since carbon emission rights do not generate cash flow, they should not be priced in a discounted manner. A random walk model should be established to price carbon options [Daskalakis G, 2009]. The GARCH model can explain the unique empirical rules of financial data and describe the financial sequence and its fluctuations through actual historical financial data [Wang Xiaofen, 2013].

Based on the current situation of the lack of pricing of carbon emission rights in China's carbon trading, studying the pricing of carbon options will help to help China's carbon products pricing in the future development of carbon trading market. However, since China has not yet opened a carbon option trading platform, the theoretical research on the pricing of carbon options in China is relatively small. Therefore, this paper discusses the pricing of carbon options in the EU carbon trading market, hoping to provide scientific pricing for the upcoming carbon options trading in China. Learn from the meaning.

CARBON OPTION PRICING MODEL

Black-Scholes model

In 1973, Professor Black and Scholes published the article entitled "Option Price and Corporate Liabilities", proposing the Black-Scholes option pricing model (hereinafter referred to as the B-S option pricing model), which caused strong repercussions in the academic and practical circles. Shortly after the birth of the B-S option pricing model, research on the relevant aspects of option pricing flourished, and the binary tree method and the finite difference method appeared successively. Since the financial asset yields are assumed to be normally distributed, the price of the asset is subject to geometric Brownian motion. The binary tree pricing

method does not limit the price change to conform to the geometric Brownian motion, so the B-S option pricing model is the pricing method of the binary tree pricing model under the special conditions of geometric Brownian motion. At present, the academic community believes that the B-S option pricing model is more simple and practical than the binary tree in the pricing of carbon emission options. Therefore, this paper selects the B-S option pricing model to study the pricing of carbon emission options.

The use of the B-S option pricing model has the following seven assumptions [Zhou Li, Li Wen, 2015]:

①The subject matter of the option is risky carbon allowance futures, whose current price is S, and assumes that carbon allowance futures such assets can be freely bought and sold;

②transaction costs and taxes are 0;

③ The option is a European option with an execution price of E and a rights period of T (in years);

④ The safe interest rate is set, that is, the risk-free rate is constant;

⑤ Before the expiration date of the option, the underlying asset carbon allowance futures have no income (such as dividends, interest, etc.), and the price changes of the options commodity are randomly distributed and continuous;

⑥The volatility of the price of carbon allowance futures is constant;

⑦ The price change of carbon allowance futures is in line with the geometric Brownian motion, that is, $dS = \mu Sdt + \sigma Sdz$.

The B-S option pricing model is derived based on the no-arbitrage principle. It considers that any option can be copied from the portfolio of underlying stocks and short-term risk-free assets. The cost of copying the portfolio is the option price. Thus they construct a portfolio of securities with a certain number of long heads and a certain number of shorts of the underlying asset, and require that the portfolio's returns are deterministic, stock-independent, and risk-free, thus deducing the following Option pricing formula:

$$C_t = S_0[N(d_1)] - Xe^{-r_c t}[N(d_2)]$$
$$d_1 = \{\ln(S_0/X) + [r_c + (\sigma^2/2)]t\}/(\sigma\sqrt{t})$$
$$d_2 = d_1 - \sigma\sqrt{t}$$
$$r_c = \ln(1 - R)$$

In the above formula, C_t represents the current value of the call option; S_t represents the current price of the underlying asset; X represents the option execution price; $N(d)$ is the cumulative probability distribution function subject to the standard normal distribution variable; r_c represents the short-term risk-free rate; t represents The number of years from the expiration date; σ^2 is the annual return rate

variance, σ represents the current price fluctuation range, table risk; R represents the risk-free continuous annual compound interest obtained by calculation; $\ln(1 - R)$ is the nature of compound interest calculation Logarithmic value.

GARCH model

Financial time series often have spikes, thick tails, and undulating clusters. Traditional econometric analysis methods cannot satisfy the same variance. Therefore, using traditional regression models to model such financial time series modeling will produce bias. In the process of studying the time series volatility (ie risk) of financial asset prices such as stock price and foreign exchange rate, scholars have explored another analytical method—autoregressive conditional heteroskedasticity (ARCH) and its extended model (GARCH). Bollerslev proposed the GARCH (1,1) model in 1986, which is the simplest GARCH model. It assumes that the conditional variance of the random error term depends to a large extent on the previous value of the error term condition variance, not just the error. The square of the previous value of the term increases the impact of the lagging volatility on itself. Therefore, it has a wider range of applications. Therefore, in the process of studying the parameter σ representing the time series volatility (ie risk) in the BS option pricing model, this paper constructs the GARCH(1,1) model and combines the predicted volatility with the BS option pricing model. The pricing method of carbon emission options based on GARCH and BS model is constructed.

The GARCH(1,1) model is expressed as follows:
$$\sigma_t^2 = \gamma V_L + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$
Where $\gamma + \alpha + \beta = 1$.

The (1,1) of the GARCH(1,1) model indicates that σ_t^2 is derived from the most recent observation of ε^2 and the latest variance. In the broader GARCH(p,q) model, σ_t^2 is the nearest observation of p ε^2 and the latest estimate of q variances. GARCH(1,1) is GARCH(p, q) The most popular of the models.

Let $\omega = \gamma V_L$, the GARCH(1,1) model can be written as:
$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$
The expression of this model is for parameter estimation. When ω, α, β are estimated, γ can be calculated from $\gamma = 1 - \alpha - \beta$, and the long-term variance $V_L = \omega/\gamma$.

In order to ensure the stability of the GARCH(1,1) model, $\alpha + \beta < 1$ is required, otherwise the weight corresponding to the long-term variance will be negative.

This simple form of setting of the GARCH (1,1) model has been widely used. With this model, market economy entities can assess future prices and risks through the previous transaction data and volatility of

the underlying assets. The variables required in the analysis are: establishing a weighted average of the long-term mean (ie, mean equation, constant), and using the lag term of the residual square of the mean equation to measure the volatility information obtained from the previous period ε_{t-1}^2 and the prediction variance σ_{t-1}^2 of the previous period. It is known that these three quantities can use GARCH(1,1) to predict the variance (volatility) of the current period. If the changes in this period are large, they will increase their expectations for the next period. The model also includes the change group in the usual financial rate of return data, and the large change in the rate of return will follow the larger change.

EMPIRICAL STUDY ON PRICING OF CARBON EMISSION OPTIONS

Sample selection

The carbon trading market under the EU ETS is the world's largest carbon trading market. The EU Emissions Quota Futures Option (EUA) is one of the major carbon emission options in the world. This product uses carbon financial futures assets as its primary assets. Therefore, the carbon emission options studied in this paper belong to carbon futures options. Considering the completeness and availability of carbon asset prices, this paper selects the daily closing price of EUA options contracts due in December 2021 during the European Energy Exchange 2018/04/03-2019/04/16 as the carbon option price. The research object of the feature test, while eliminating the transaction data with zero closing price due to no volume during this period, a total of 271 data were obtained. All data in this article is from https://www.eex.com/en.

Parameter Estimation of B-S Option Pricing Model

(1) The number of years from the option to maturity (t)

The expiration period of the carbon option refers to the period of time that the carbon option holder can hold the carbon option when the relevant contract is formulated. The general term is an integer and is the end of the year of the expiration. The EUA options contract for the European Energy Exchange selected in this paper is 5 years old and expires in December 2021,t=(the number of options from the due date)/365.

(2) Short-term risk-free interest rate (r_c)

The short-term risk-free rate refers to the interest rate that can be obtained by investing funds in an investment without any risk. This is an ideal investment income. The European banking industry chose the Euro short-term interest rate (ESTER) as a risk-free rate, so the current ECB's latest deposit-

facilitating rate of -0.4% is used as a short-term risk-free rate.

(3) Volatility (σ)

Through the pre-processing of the carbon yield option price yield series, it is proved that the sequence can be predicted by the GARCH model, and then the GARCH(1,1) model is established and its stationarity is proved. The next section will use the GARCH(1,1) model prediction formula to derive volatility over the next 20 days.

(4) Option execution price (X)

The strike price of a carbon option is the price actually paid by the holder of the carbon option at maturity. The execution price of the option in this article is the median of the annual transaction data, $X = 21.37$.

(5) The price of the underlying asset of the option (S_0)

The EUA carbon emission option studied in this paper belongs to the carbon futures option, so the underlying asset price is the carbon futures price, $S_0 = 26.95$.

Calculate volatility using the GARCH model

First, the data is pre-processed, and descriptive statistical analysis, unit root test, and sequence autocorrelation and partial autocorrelation test are performed on the price-earning rate series of carbon emission options. Then the volatility model is established and the residual sequence is tested for ARCH effects. Finally, based on the conditional heteroskedasticity of the residual sequence, the GARCH(1,1) model is established and the model is tested and predicted. All steps are performed on the Eviews 6.0 statistical software.

In order to make the test better, this paper uses Eviews to logarithmize the daily settlement price data of EUA options, and obtain the logarithmic rate of return, that is, $R = \log（y_i/y_{i-1}）$.

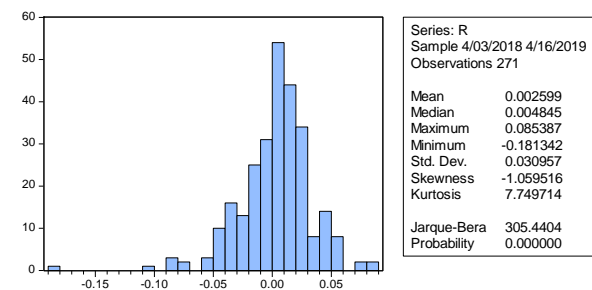(1) Descriptive statistical analysis

Figure 1 Descriptive statistical analysis results of the EUA option daily closing price sequence

① The sequence skewness is -1.059516, which belongs to the left bias. The kurtosis value is equal to 7.774714, which is much larger than the K value of the normal distribution. The degree of convexity of the distribution is greater than the normal distribution,

indicating that it has obvious characteristics of "spike and thick tail".

② The JB statistic is used to test whether the sequence observations obey the normal distribution. The null hypothesis of the test is that the sample follows a normal distribution. The JB statistic of this sequence is equal to 305.4404, and the critical value above the 5% significance level is 5.99. The null hypothesis is rejected, indicating that the sequence does not obey the normal distribution, and the income distribution is initially considered to have a "thick tail" feature.

③The P value is the probability of rejecting the Type I error made by the null hypothesis. The P value is 0, so the null hypothesis can be rejected at the 1% significance level, ie the sequence does not conform to the normal distribution.

(2) Unit root test

Before performing time series analysis, its stationarity must be determined. The most popular test method at present is the unit root test. In view of the better performance of the ADF, the ADF method is used to test the stability of the sequence. The details are shown in Figure 2.

Null Hypothesis: R has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 3 (Automatic based on AIC, MAXLAG=15)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.101066	0.0000
Test critical values:	1% level	-3.992801	
	5% level	-3.426745	
	10% level	-3.136628	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(R)
Method: Least Squares
Date: 04/19/19   Time: 10:52
Sample (adjusted): 4/09/2018 4/16/2019
Included observations: 267 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
R(-1)	-1.040699	0.114349	-9.101066	0.0000
D(R(-1))	0.102276	0.098838	1.034781	0.3017
D(R(-2))	0.180270	0.083552	2.157566	0.0319
D(R(-3))	0.116052	0.061550	1.885483	0.0605
C	0.003864	0.003881	0.995513	0.3204
@TREND(4/03/2018)	-7.75E-06	2.46E-05	-0.315564	0.7526

R-squared	0.477814	Mean dependent var	-7.17E-05
Adjusted R-squared	0.467811	S.D. dependent var	0.042373
S.E. of regression	0.030911	Akaike info criterion	-4.093164
Sum squared resid	0.249390	Schwarz criterion	-4.012552
Log likelihood	552.4374	Hannan-Quinn criter.	-4.060783
F-statistic	47.76447	Durbin-Watson stat	2.008373
Prob(F-statistic)	0.000000		

Figure 2 ADF unit root test results of the EUA option daily closing price sequence

The price-earnings ratio of the carbon emission option is obtained by the ADF unit root test. The value of τ is equal to -9.101066, and the corresponding p-value is close to 0, indicating that the sequence rejects the non-stationary hypothesis, that is, there is no unit root, and the sequence {r} is stable.

(3) Sequence autocorrelation and partial autocorrelation test

After verifying that the carbon yield option price return sequence is a stationary sequence, the autocorrelation and partial autocorrelation of the sequence are further tested. The residual sequence is obtained after the mean value regression of the carbon yield option price return rate series, and the residual sequence and the residual square sequence are subjected to a 12-step autocorrelation and partial autocorrelation test respectively. The details are shown in Figure 3.

Date: 04/19/19   Time: 10:55
Sample: 4/03/2018 4/16/2019
Included observations: 271

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.069	0.069	1.3209	0.250
		2	0.068	0.064	2.5989	0.273
		3	-0.063	-0.072	3.6885	0.297
		4	-0.118	-0.115	7.5446	0.110
		5	-0.051	-0.027	8.2604	0.142
		6	-0.065	-0.049	9.4366	0.150
		7	-0.063	-0.068	10.559	0.159
		8	0.080	0.080	12.369	0.135
		9	0.063	0.050	13.499	0.141
		10	0.037	-0.003	13.889	0.178
		11	-0.031	-0.051	14.159	0.224
		12	0.001	0.021	14.160	0.291

Figure 3 Autocorrelation and partial autocorrelation test results of the EUA option daily closing price series

The autocorrelation and partial autocorrelation coefficients of the sequence fall within twice the estimated standard deviation, and the corresponding p-values of the Q statistic are greater than the confidence level of 0.05, so the sequence has no significant correlation at the 5% significance level.

(4) Establishing a volatility model

Since there is no significant correlation between the carbon yield option price yield series, the mean equation is set to white noise.

Set up the model: $r_t = \mu + \varepsilon_t$, where μ is a constant term and ε_t is the error term.

The r_t is de-averaged to obtain $w = r_t - 0.002599$, and the descriptive statistics of w are shown in Figure 4.

Series: W
Sample 4/03/2018 4/16/2019
Observations 271

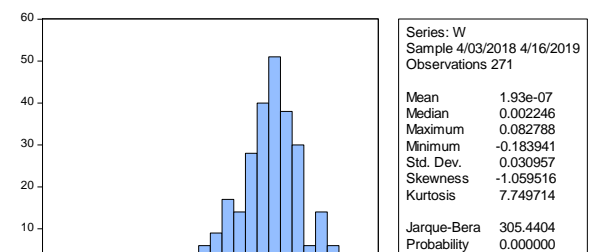|	
Mean	1.93e-07
Median	0.002246
Maximum	0.082788
Minimum	-0.183941
Std. Dev.	0.030957
Skewness	-1.059516
Kurtosis	7.749714
Jarque-Bera	305.4404
Probability	0.000000

Figure 4 Descriptive statistical analysis results after the average closing of the EUA option daily closing price

(5) ARCH test

First establish the square equation of w z=w^2

Then get the autocorrelation function analysis graph:

Date: 05/06/19 Time: 21:59
Sample: 4/03/2018 4/16/2019
Included observations: 271

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.093	0.093	2.3571	0.125
		2	0.060	0.052	3.3366	0.189
		3	0.157	0.148	10.098	0.018
		4	0.190	0.167	20.097	0.000
		5	0.029	-0.011	20.330	0.001
		6	-0.038	-0.081	20.732	0.002
		7	-0.016	-0.066	20.804	0.004
		8	0.035	0.011	21.143	0.007
		9	0.026	0.046	21.341	0.011
		10	0.015	0.048	21.409	0.018
		11	0.004	0.009	21.413	0.029
		12	-0.036	-0.065	21.779	0.040

Figure 5 ARCH test results of the EUA option daily closing price sequence

According to Figure 3-5, at the 3rd order of lag, the adjoint probability of the ARCH test is less than the significance level of 0.05, rejecting the null hypothesis, the residual sequence has heteroscedasticity, and there is an ARCH effect. In the conditional heteroscedasticity theory, when the lag term is too much, it is appropriate to use the GARCH(1,1) model instead of the ARCH model, which also shows the rationality of using the GARCH(1,1) model.

(6) Establish GARCH (1,1) model

Dependent Variable: W
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 05/10/19 Time: 19:58
Sample (adjusted): 4/04/2018 4/16/2019
Included observations: 270 after adjustments
Convergence achieved after 18 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.098424	0.071441	1.377707	0.1683
Variance Equation				
C	1.93E-05	1.20E-05	1.614488	0.1064
RESID(-1)^2	0.114859	0.016708	6.874397	0.0000
GARCH(-1)	0.874850	0.021835	40.06636	0.0000
R-squared	0.003981	Mean dependent var		1.73E-06
Adjusted R-squared	0.003981	S.D. dependent var		0.031014
S.E. of regression	0.030952	Akaike info criterion		-4.238989
Sum squared resid	0.257714	Schwarz criterion		-4.185679
Log likelihood	576.2635	Hannan-Quinn criter.		-4.217582
Durbin-Watson stat	2.068749			
Inverted AR Roots	.10			

Figure 6 GARCH(1,1) model diagram of the EUA option daily closing price sequence

The established GARCH(1,1) model has a small p-value, that is, both ARCH and GARCH are highly significant, indicating that the data has a strong volatility agglomeration effect, and the AIC and SC values in the model are relatively small. The data is fitted to the ground. The D-W statistic is approximately equal to 2, indicating that the yield conditional variance sequence is stationary, there is no autocorrelation, and the model's estimation result is good.

The model established is as follows:

Mean equation: $r_t = 0.098424r_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma_t^2)$

Variance equation: $\sigma_t^2 = 0.0000193 + 0.114859\varepsilon_{t-1}^2 + 0.874850\sigma_{t-1}^2$ Where $\alpha_0 = 0.0000193 > 0, \alpha > 0, \beta > 0$, and the constraint condition of GARCH(1,1) model construction is satisfied, $\alpha + \beta = 0.114859 + 0.874850 = 0.989709 < 1$, indicating that the volatility model is stationary.

According to the prediction formula of the GARCH(1,1) model, the volatility of the next 20 days is predicted, as shown in Table 1:

Table 1 Volatility Table for EUA Options in the Next 20 Days

Date	Predicted volatility	Date	Predicted volatility
1	12.51%	11	19.61%
2	13.42%	12	20.16%
3	14.26%	13	20.69%
4	15.05%	14	21.21%
5	15.80%	15	21.71%
6	16.51%	16	22.20%
7	17.18%	17	22.67%
8	17.82%	18	23.13%
9	18.44%	19	23.58%
10	19.04%	20	24.02%

It can be seen that the predicted volatility in the next 20 days is gradually increasing, indicating that the GARCH (1,1) model also has some limitations, that is, the error of the volatility prediction result will become larger as time goes by. Therefore, the GARCH(1,1) model is suitable for short-term volatility prediction.

EMPIRICAL RESULTS

Substituting the above estimated parameters into the formula of the B-S option pricing model, the predicted price of the carbon emission option in the next month is obtained, as shown in Table 2:

Table 2 Comparison of predicted and actual prices of EUA options in the next 20 days

Date	Forecast price of EUA	Actual price of EUA options
1	5.704301	28.20
2	5.78733	27.63
3	5.869597	27.63
4	5.950569	27.63
5	6.029952	28.28
6	6.107596	28.14
7	6.183434	28.02
8	6.257453	26.59
9	6.329668	27.07
10	6.400116	26.96
11	6.468846	26.96
12	6.535909	25.31
13	6.601364	25.82
14	6.665266	25.98
15	6.727674	27.03

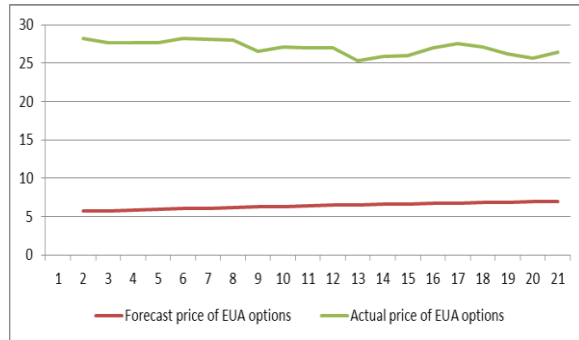| 16 | 6.788643 | 27.55 |
17	6.848226	27.14
18	6.906475	26.18
19	6.96344	25.60
20	7.019168	26.43

Figure 7 Comparison of the predicted price and actual price of EUA options in the next 20 days

In this paper, the price of EUA options in the next 20 days calculated by using GARCH model and BS option pricing model fluctuates between 5.704301 and 7.019168 euros, and shows a slow upward trend. The actual price range of EUA options during this period ranges from 25.31 to 28.28 euros. The difference between the calculated price and the calculated price can explain that the GARCH model and the B-S option pricing model can enable the company to conduct market transactions at a lower cost, greatly reducing the transaction risk, and the company has more cash flow for the enterprise. The daily production can further promote the enthusiasm of the company.

However, there are still some shortcomings in applying the B-S option pricing model to the pricing of carbon options. First of all, the B-S option pricing model used in this paper has a series of assumptions, which are too idealistic and may not be inconsistent with the actual situation. Second, over time, the GARCH (1,1) model predicts that the volatility error will become larger and larger, which is unfavorable for the pricing of carbon options. Finally, although the selected time period is considered representative, it is not the whole data of the EUA option contract transaction, and it cannot fully represent the EUA option trading under the EU ETS. Although there are several shortcomings mentioned above, this paper verifies that the pricing methods based on GARCH and B-S models can be applied to the pricing of carbon options, at least for the pricing of carbon options in China.

CONCLUSIONS

In order to establish and improve China's unified carbon emission trading market and solve the problems existing in China's current carbon emission pricing, this paper firstly combs the research literature on carbon option pricing at home and abroad, introduces the necessity of carbon option pricing and carbon options. The method of pricing; then the descriptive statistical analysis, unit root test, sequence autocorrelation test and ARCH effect test of the carbon emission option price yield series prove the use of GARCH (1,1) model to predict the volatility The rationality of the EU emission allowance futures options under the EU ETS is based on the GARCH and BS model pricing methods. Through the above analysis and research, the main conclusions of this paper are summarized as follows:

(1) Based on the current situation of China's lack of pricing power in carbon emission trading, it is very necessary to study carbon option pricing. As a carbon financial derivative, carbon options can hedge the uncertainty of future carbon emission allowance prices while meeting emission reduction targets. At the same time, using carbon options to circumvent price risks can impose restrictions on imported carbon emissions.

(2) The GARCH model can meet the needs of features such as spikes, thick tails and undulating clusters in financial time series. The traditional econometric analysis method cannot meet the requirements of the same variance. Therefore, using traditional regression models to model such financial time series modeling will produce bias. The GARCH model can explain the unique empirical rules of financial data, and can describe the typical characteristics of financial sequences and their fluctuations through actual historical financial data.

(3) Although the B-S option pricing model is flawed, it can be used in the pricing of carbon options. A series of assumptions in the B-S option pricing model are too idealistic, which is inconsistent with the actual situation of carbon options. Many scholars have also improved the B-S option pricing model. This paper combines the GARCH model with the B-S option pricing model, which improves the accuracy of carbon option pricing results to a certain extent, and can provide reference for the research of carbon option pricing.

REFERENCES

Chen Ting. Analysis of economic effects of environmental tax and research on environmental tax construction in China [D]. Anhui University, 2012.

Corjan Brink,Herman R.J. Vollebergh,Edwin van der Werf. Carbon pricing in the EU: Evaluation of different EU ETS reform options[J]. Energy Policy,2016,97.

George Daskalakis,Dimitris Psychoyios,Raphael N. Markellos. Modeling CO 2 emission allowance prices

and derivatives: Evidence from the European trading scheme[J]. Journal of Banking and Finance,2009,33(7).

Huang Ping, Wang Yulu. Research on the Formation of Carbon Emission Price in China——Based on the Value Network Analysis of CDM Projects[J]. Price Theory and Practice, 2010(08): 24-25.

Ivan Diaz-Rainey,Daniel J. Tulloch. Carbon pricing and system linking: Lessons from the New Zealand Emissions Trading Scheme[J]. Energy Economics,2018,73.

Li Xu,Shi-Jie Deng,Valerie M. Thomas. Carbon emission permit price volatility reduction through financial options[J]. Energy Economics,2016,53.

Liu Xubin. Research on option pricing based on neural network method [D]. Jinan University, 2011.

Marc S. Paolella,Luca Taschini. An econometric analysis of emission allowance prices[J]. Journal of Banking and Finance,2008,32(10).

Shi Shengzhen, Huang Tongcheng. Option Mechanism: A New Attempt of Emission Trading[J]. Productivity Research, 2005(03): 60-62.

Shu Xin, Deng Xiaowei. Research on the volatility of spot and forward rate of return in carbon market [J]. Green Technology, 2018 (14): 129-131.

Wang Qiupu. Theoretical Analysis of Carbon Assets Evaluation under the Background of Environmental Finance [D]. Jinan University, 2015.

Wang Xiaofen. Study on pricing of carbon emission rights [D]. East China Jiaotong University, 2013.

Wu Hongjie. Carbon Asset Management [M]. Beijing: Beijing United Publishing Company, 2015: 3-4.

Yang Jiawei. Analysis of International Carbon Trading Market and Its Derivative Financial Products[J].Financial Finance,2009(2):133-134.

ZHANG Chen, PENG Ting, LIU Yujia. Research on carbon option pricing based on GARCH-fractal Brownian motion model[J].Journal of Hefei University of Technology(Natural Science),2015,38(11):1553-1558.

ZHAO Xiaopan, LI Chaohong, REN Xiaoge. Pricing of Carbon Emission Rights Based on Black-Scholes Option Pricing Model[J].Commercial Accounting,2016(07):28-31.

Zhou Li, Li Wen, Mai Xin. Carbon Asset Pricing Technology and Method [M]. Guangdong: South China University of Technology Press, 2015: 270.

Zhu Yuexi, Chen Hongxi, Zhao Zhimin. Research on Pricing of Carbon Emissions Trading Based on B-S Pricing Model[J].Science & Technology Progress and Policy,2013,30(05):27-30.